Foundations of Physics, Vol. 12, No. 10, 1982

The de Broglie Pilot Wave Theory and the Further Development of New Insights Arising Out of It

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Received April 16, 1982

We briefly review the history of de Broglie's notion of the "double solution" and of the ideas which developed from this. We then go on to an extension of these ideas to the many-body system, and bring out the nonlocality implied in such an extension. Finally, we summarize further developments that have stemmed from de Broglie's suggestions.

1. INTRODUCTION

Originally, Schrödinger regarded the wave intensity, $\psi^*\psi$, as the actual density of electric charge, and this gave an image of the electron as a unique and independent reality. However, Schrödinger's equation itself, because of its linearity, implied that this assumed charge density would spread out without limit in a short time; and yet, the particle is always found in a small region of space. To resolve this difficulty, Born proposed that the wave intensity is the probability density of *finding* a particle. However, this left open the question of whether the localized particle exists independently, or whether it is in some sense produced or at least localized in the act of observation (as is indeed implied in Heisenberg's analysis of the measurement of position and momentum).

It is well-known that in the usual interpretation of the quantum theory, the latter point of view is adopted. The most consistent version of this interpretation is that given by Bohr,(1) in which no meaning can be ascribed to precisely defined particle properties beyond the limits specified by the

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0015-9018/82/1000-1001$03.00/0

Heisenberg uncertainty principle. This lack of meaning goes beyond regarding these properties as independently existent, but uncertain *to us* because of limits in our ability to obtain knowledge of them through measurements. Rather, in Bohr's view, the universe is basically an unanalyzable whole, in which the notion of the separateness of particle and environment is an abstraction that has no content, except as an approximation that may be applied within the limit of Heisenberg's principle.

Most physicists have adopted views similar in key ways to those described above, though the details vary considerably. However, de Broglie, Einstein, Schrödinger, and others disagreed with this approach, because they felt that there is a uniquely defined reality, which can be grasped in thought and is yet independent of thought. Without considering this reality, science is reduced to a set of formulas and recipes for predicting the results of experiments. Indeed, a large number of modern physicists have since then, at least tacitly, come to adopt such a point of view, perhaps because it is part of the pragmatic spirit of the age. However, in our view, this pragmatic approach is not the only possible one, nor is it even the best. For one can see that concepts that do not give immediate new experimental predictions may still be valuable, in that they permit new insight and understanding (from which new predictions may ultimately emerge). An approach that discourages this kind of insight will thus tend to prevent creative new perceptions, such as those of Einstein, de Broglie, etc.

The pilot wave theory of de Broglie[2] was indeed a significant and fruitful example of imaginative concepts that help lead to new insights. We begin with a brief historical summary of this idea and its development. Of course, in such a short space, we cannot hope to given a complete account, but rather, we select a few points that are relevant to our discussion.

In essence, de Broglie assumed that there is a physically real wave satisfying Schrödinger's equation, at least as a linear approximation, along with a particle following a well defined trajectory. The momentum of this particle was related to the wave through the equation

$$\mathbf{p} = \hbar \nabla \Phi \tag{1}$$

where ϕ is the phase of the wave function. The above relationship suggests that the particle is being "guided" by the background wave, and for this reason, de Broglie called the latter a "pilot wave". (One may here consider the analogy of an airplane guided by radar waves, which carry information about the whole environment).

De Broglie gave a particularly beautiful explanation of how the pilot wave would actually guide the particle. He proposed that inside the particle was a periodic process that was equivalent to a clock. In the rest frame, the clock would have a frequency, $\omega_0 = m_0 c^2/\hbar$. By considering how this clock would behave in other frames, he derived what is in essence the Bohr–Sommerfeld relationship

$$\oint \mathbf{p} \cdot \mathbf{d}x = nh$$

which is the condition for the clock to remain in phase with the pilot wave.

One may here think of the phase locking of synchronous motors through a nonlinear interaction. A similarly nonlinear interaction would of course be needed to lock the phase of the clock to that of the pilot wave. This sort of interaction was indeed implied in de Broglie's further development of the model. What he suggested in addition was that nonlinearity of the played a crucial role, in that it made possible a singular solution, representing the particle, which had to fit smoothly onto a weak background field, obeying Schrödinger's equation as a linear approximation. This requirement of a smooth connection of the two members of the "double solution" was, of course, what explained the guidance condition. It is significant to note here that his model provides at least a conceptual connection between quantum mechanics and Einstein's attempt at a unified field theory, in which the particle was also treated as a nonlinear singularity that merges with a linear background field (modern soliton theory is also closely related in concept to this approach).

The fate of the theory was decided at the Solvay Conference, where Pauli[3] made important objections to it, based on the fact that the theory did not provide a consistent account of the many body system. (in particular, he discussed a general two-body scattering process.) While de Broglie felt that he had at least the germ of an answer, this was apparently not appreciated by the others who were present. This, along with the fact that not even Einstein spoke up for the theory, led to a definite rejection, which was indeed so decisive that de Broglie himself gave up work on the theory.

However, in 1952, one of us (Bohm[4,5]) published two papers on the subject.[2] In the first of these, the consequences of what was in essence de Broglie's theory were worked out in considerable detail, and shown to provide a generally consistent account of quantum mechanics for the one-body system. In the second paper, this work was extended to the many-body system, in a way that answered Pauli's objections, and made possible certain key new insights into the meaning of the quantum mechanics. This encouraged de Broglie to take up his idea again, ultimately to propose a theory, in which the particle is assumed to be in a "thermal" bath, provided

[2] See also the Appendix to present paper.

by a background of vacuum fluctuations. Since then, a number of others (some of which will be discussed in Section 3) have followed along these lines, though, of course, it must be admitted that this approach is still not accepted by most physicists (see for example Bohm and Vigier,[6] Vigier,[7] Nelson,[8] de la Pena[9] and Roy.[10])

2. TRAJECTORY INTERPRETATION FOR A MANY-BODY SYSTEM

We now give an account of how the many-body system is to be understood in terms of an extension of the ideas of de Broglie's, which were discussed in the previous section.

The basic starting point is to write the N body wave function, as

$$\psi(\mathbf{x}_1 \cdots \mathbf{x}_n) = R(\mathbf{x}_1 \cdots \mathbf{x}_n) \exp[iS(\mathbf{s}_1 \cdots \mathbf{x}_n)/\hbar]$$

and to defined the momentum of the nth particle as

$$\mathbf{p}_n = \nabla_n S \tag{2}$$

(which is evidently a generalization of de Broglie's relation (1)). By substituting (2) into the many-body Schrödinger equation, we then obtain the conservation equation in configuration space

$$\frac{\partial P}{\partial t} + \sum_n \nabla_n \cdot \frac{(P\nabla_n S)}{m} = 0 \tag{3}$$

(where $P = \Psi^* \Psi$ is the probability density in this space), and the modified Hamilton–Jacobi equation

$$\frac{\partial S}{\partial t} + \sum_n \frac{(\nabla_n S)^2}{2m} + V(\mathbf{x}_1 \cdots \mathbf{x}_n) + Q(\mathbf{x}_1 \cdots \mathbf{x}_N) = 0 \tag{4}$$

From this, it follows that each particle will be acted on, not only by the classical potential, V, but also by an additional quantum potential

$$Q = \frac{-\hbar^2}{2m} \sum_n \frac{(\nabla_n^2)R}{R} \tag{5}$$

In this interpretation, the new features of quantum mechanics are seen to arise basically from Q. To illustrate how this comes about, let us begin by considering the special case of a two-body system, with a product wave function

$$\Psi(\mathbf{x}_1, \mathbf{x}_2) = \phi_A(\mathbf{x}_1)\,\phi_B(\mathbf{x}_2) \tag{6}$$

Defining

$$\phi_A(\mathbf{x}) = R_A(\mathbf{x})\,e^{iS_A(\mathbf{x})/\hbar}$$

and (7)

$$\phi_B(\mathbf{x}) = R_B(\mathbf{x})\,e^{iS_B(\mathbf{x})/\hbar}$$

we can see immediately that

$$Q = \frac{-\hbar^2}{2m}\frac{\nabla_1^2 R_A(\mathbf{x}_1)}{R_A(\mathbf{x}_1)} - \frac{\hbar^2}{2m}\frac{\nabla_2^2 R_B(\mathbf{x}_2)}{R_B(\mathbf{x}_2)} \tag{8}$$

The above is the sum of two independent functions. If the classical potential V is likewise a sum, $V_A(\mathbf{x}_1) + V_B(\mathbf{x}_2)$, it follows that the Hamilton–Jacobi equation reduces to two separate parts

$$\frac{\partial S_A}{\partial t} + \frac{(\nabla_1 S_A)^2}{2m} + V_A(\mathbf{x}_1) - \frac{\hbar^2}{2}\frac{\nabla_1^2 R_A(\mathbf{x}_1)}{R_A(\mathbf{x}_1)} = 0 \tag{9a}$$

$$\frac{\partial S_B}{\partial t} + \frac{(\nabla_2 S_B)^2}{2m} + V_B(\mathbf{x}_2) - \frac{\hbar^2}{2m}\frac{\nabla_2^2 R_B(\mathbf{x}_1)}{R_B(\mathbf{x}_2)} = 0 \tag{9b}$$

Evidently, the conservation equation also splits into two independent parts.

We see then that the one-body equation (as treated by de Broglie) arises as an abstraction and a simplicication of that of the two-body system, and eventually of the N-body system. (It is clear moreover than ultimately these N-bodies must be extended to include the whole universe.)

One of the most important ways in which this interpretation gives new insight is that it enables us to express quantum mechanics and classical mechanics in terms of the same language, so that we can see their similarities and their differences more clearly than is possible in the usual approach, in which they are treated in terms of very different modes of description.

The first main difference can be seen by noting that the quantum potential, Q, is not altered when the wave function is multiplied by a constant, so that it does not fall to zero at long distances, where the wave intensity becomes negligible. However, the classical notion of analyzability of a system into independent parts depends critically on the assumption that whenever the parts are sufficiently far removed from each other, they do not significantly interact. This means that the quantum theory implies a new kind of wholeness, in which the behavior of a particle may depend significantly on distant features of the over-all environment. This dependence produces consequences similar to those implied by Bohr's notion of unanalyzable wholeness, but different in that the universe can be understood as a unique and in principle well defined reality.

To illustrate in more detail what is meant here, we consider an interference experiment, in which a beam of electrons of definite momentum is sent through a two slit system. In Fig. 1, we show the results of a computation of the quantum potential[11]; and in Fig. 2, we show the trajectories resulting from the potential.

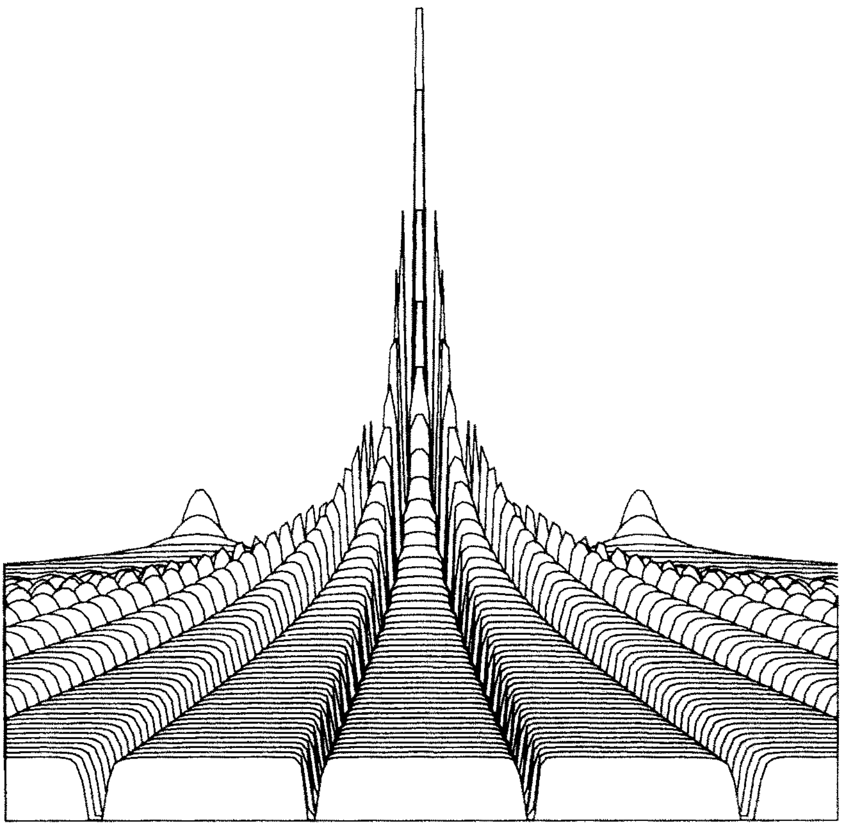

Fig. 1. Quantum potential for a pair of Gaussian slits. The slits can be seen in the background. The fringes are formed in the foreground, the dark bands coinciding with the valleys of the quantum potential.

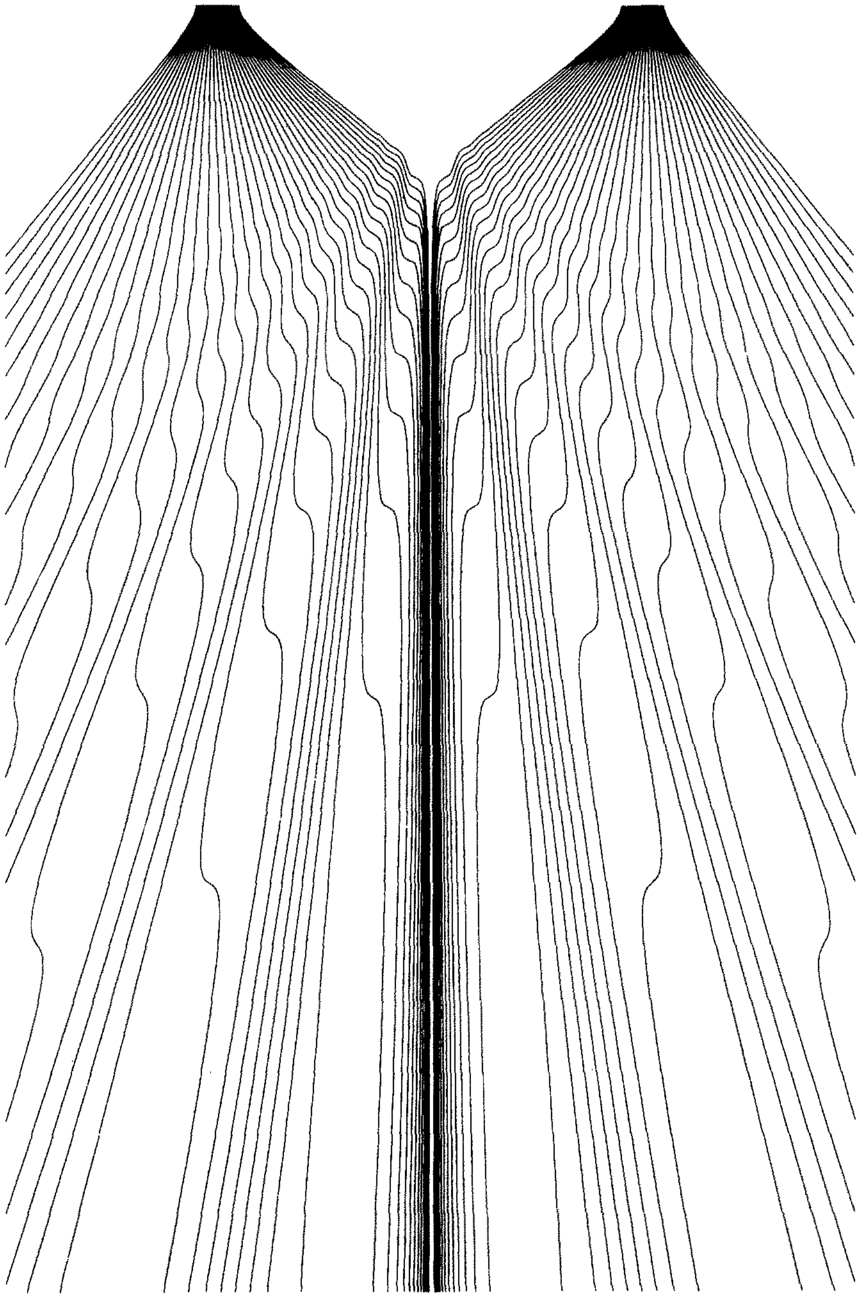

Fig. 2. The particle trajectories emanating from the Gaussian slits on the left-hand side of the figure. The fringes on the right result from the bunching of the trajectories.

What is especially significant in Fig. 1 is that the quantum potential remains large at long distances from the slits, taking the form of a set of valleys and high ridges, which latter gradually flatten out into broad plateaux. In Fig. 2, one sees how the trajectories are ultimately bunched into these plateaux by the over all effect of the potential, and that this brings about the interference pattern. (So that, for example, if one of the slits had been closed, the quantum potential would have been a smooth parabolic function, which would produce no pattern of fringes). The fact that the quantum potential does not in general fall off with the distance is thus what explains interference and diffraction patterns, and this is clearly also what implies the kind of wholeness of particle and environment to which we have referred above.

One may return here to the analogy of the airplane guided by radar waves. Evidently, it is not a case of mechanical pressure of these waves on the airplane, but rather, information concerning the whole environment is enfolded by the waves, and carried into each region of space. The airplane thus responds actively to the *form* of the waves, and this form is not altered as the intensity falls of with the distance. A similar response to the *form* of the quantum potential is seen to be characteristic of the behavior of the electron. This means that in the microworld, the concept of active information is relevant (see Bohm and Hiley[12] for more detail).

What has been said thus far about the new kind of wholeness implied by the quantum theory for the one-body system is further strengthened by a consideration of the many-body system. For here, one finds that when the wave function is no longer separable as a product of functions of the coordinates of each particle, the quantum potential leads to a strong interaction between all the particles of the system, that does not in general fall off to zero when the particles are distant from each other. There is evidently an extension of the dependence of the particle on its over all environment that characterizes the one-body system. But in addition, there is a yet more thoroughgoing breakdown of the possibility of analysis, because the force acting on each particle is no longer expressible as a predetermined function of the positions of the other particles. Rather, the functional form of the force depends on the whole set of conditions in which the wave function is defined and determined. (So that, for example, the form changes whenever this quantum state of the whole changes.)

Let us take, as an example, the hypothetical experiment of Einstein, Podolsky, and Rosen.[13] We consider there the original form of the experiment, in which we start with a quantum state of a two-particle system in which $x_1 - x_2$ and $p_1 + p_2$ are both determined. This is given by

$$\Psi(x_1 x_2) = f(x_1 - x_2 - a) = \sum_k C_k \exp[ik(x_1 - x_2 - a)] \tag{10}$$

where $f(x_1 - x_2 - a)$ is a packet function, sharply peaked at $x_1 - x_2 = a$, while C_k is its Fourier coefficient. Evidently, in this state, $p_1 + p_2 = 0$, while $x_1 - x_2$ can be made as well defined as we please.

In this experiment, one can measure x_1 and immediately know that $x_2 = x_1 + a$ (to an arbitrarily high degree of accuracy). Alternatively, we can measure p_1 and immediately know that $p_2 = -p_1$. In both cases, the first particle is disturbed in the process of measurement and, of course the disturbances can account for the Heisenberg uncertainty relations as applied to the particle $\Delta p_1 \, \Delta x_1 \geqslant h$. But since the second particle is assumed not to interact with the first in any way at all, it follows that we are able to find its properties without its having undergone any disturbance whatsoever. Nevertheless, according to the quantum theory, the uncertainty principle, $\Delta p_2 \, \Delta x_2 \geqslant h$ must still apply. So Heisenberg's explanation of this uncertainty as due to a disturbance resulting from measurement can no longer be used. It was this which indeed led Einstein, Podolsky, and Rosen[13] to argue that since both x_2 and p_2 were in principle measurable to arbitrary accuracy without a disturbance,vthey must have already existed independently in particle 2 as 'elements' of reality with well defined values before the measurement took place. And so, they concluded that quantum mechanics is an abstraction giving only an incomplete and fragmentary description of the underlying reality (as insurace statistics are abstractions that similarly yield an incomplete and fragmentary description of the people to whom they are applied).

As is well-known, Bohr[14] answered this argument by means of a further development of his notion that the measurement process is an unanalyzable whole, which led in this case to the conclusion that there is no meaning to the attempt to given a detailed description of how correlations of position and momentum are carried along by the movements of the parts of a many-body system. It is interesting, however, to go carefully into how the trajectory interpretation differs from that of Bohr, and yet comes to a similar notion of unanalyzable wholeness, though, of course, in another way. For this case, writing $f = Re^{iS/\hbar}$, we obtain for the quantum potential

$$Q = \frac{-\hbar^2}{2m} \left(\frac{\partial^2 R}{\partial x_1^2} + \frac{\partial^2 R}{\partial x_2^2} \right) \Big/ R = \frac{-\hbar^2}{m} \frac{\partial^2 R}{\partial \Delta x^2} (\Delta x - a) \Big/ R(\Delta x - a) \qquad (11)$$

with $\Delta x = x_1 - x_2$. This function evidently remains large, even when the distance, a, separating the particles is not small. Therefore, when the properties of the first particle are measured, the quantum potential brings about a corresponding disturbance of the second particle. And from this, it can be shown[5] that in a statistical ensemble of similar measurements, Heisenberg's uncertainty solutions, $\Delta p_2 \, \Delta x_2 \geqslant h$ will still be obtained.

If follows then that with the aid of the quantum potential, something like Heisenberg's original explanation of the uncertainty principle can be maintained. The uncertainties in the properties of particles are indeed now seen to follow from disturbances produced by the quantum potential, whose effects are moveover unpredictable and uncontrollable, because the current form of the theory is only able to provide information concerning statistical distributions over the initial conditions of all the particles concerned. (So that we cannot in practice make observations of individual particles going beyond Heisenberg's principle.)

This, however, raises the interesting question as to whether quantum uncertainties in general represent nothing more than our inability to obtain complete information about what is in itself an in principle determinate individual system, or whether there is not some kind of further indeterminism that is inherent in the individual system itself. The answer to this question is rather subtle, and requires that we go more carefully into the feature of unanalyzable wholeness to which we have already called attention. In the case of the EPR experiment, this wholeness is shown by the fact that interaction between the two particles depends on the wave function of the whole system from which, as we have seen, it follows that in different quantum states, this interaction may be different. In principle, this dependence of the interaction on the state of the whole should extend to include the entire universe (See Bell,[15] who also uses the wave function of the universe but in another way). However, for suitable large-scale systems, one can see there is in general a classical limit, in which the wave function approximately factorizes so that the forces between particles in each system may be referred only to the state of that system alone.[12] But any such treatment in terms of large-scale systems is evidently a gross simplification of a much more complex and subtle process. For example, several quantum systems may separate in such a way that their wave functions factorize, so that they behave independently. Yet, later, such systems may combine again with each other and with other systems to form new quantum states of a single larger system. When this happens, the systems that were independent now cease to be so, and a new quantum potential arises, in which the interactions between particles within any one of the initially independent systems are now found to be dependent on the state of the whole larger system in which they take part.

From the above, it is clear that in the long run only the entire universe can be regarded as self determinate while any part may be independent in general only for some limited period of time. This is not only in the rather superficial classical sense that the positions and momenta of its various constituent parts of a given system may be altered in interaction with other systems. Rather, it is in the more fundamental sense that the very mode of interaction between constituent parts depends on the whole, in a way that cannot be specified without first specifying the state of the whole. Therefore the simple notion of mechanical determinism no longer applies, but rather, something much more subtle is involved (for further developments along these lines see Bohm and Hiley[12]).

Of course, it must be pointed out that this theory is thus far inherently nonrelativistic, because the quantum potential implies an instantaneous connection between distant particles. De Broglie, like Einstein, therefore did not feel that the extension to the many-body system as proposed here, provided a generally satisfactory treatment. We shall discuss this issue in more detail in the next section, but broadly speaking, there are two possible approaches that may be adopted. In the first, one accepts relativity (in its field theoretical form) as being a generally correct starting point, and tries from there to explain the many-body quantum wave equation by means of further particular assumptions within this framework. This is, of course, the attitude that would be favored by Einstein and de Broglie. In the second approach, however, one regards the basic concepts of both relativity and quantum theory as abstractions from, and limiting cases of, a more general conceptual framework. In particular, we have proposed for the framework a new notion of implicate order. We shall not, however, go further here into this notion, which is treated in detail elsewhere.[16]

3. FURTHER DEVELOPMENTS FROM DE BROGLIE'S IDEAS

One of the first developments after the work described in Section 2 was to explain the probability distribution, $P = \psi^*\psi$ (which had thus far just been demonstrated to be consistent with the conservation equation). This was done in terms of a hydrodynamical model by Bohm and Vigier.[6] A fluid was assumed, along the lines originally proposed by Madelung,[17] whose mean density was $P = \psi^*\psi$, and whose mean current was $\mathbf{j} = \psi^*\psi \, \nabla S/m$. In addition, it was supposed that a particle was being carried along the flow lines of the fluid. But the fluid was further assumed to be a deeper "sub-quantum-mechanical" medium, whose inner structure was in random movement, which was communicated to the particle to produce Brownian motion. It was then shown that for a wide variety of assumptions concerning this random motion, an arbitrary initial distribution of probability, $P_0(x)$ was transformed eventually into $P = \psi^*\psi$, and that this was therefore in effect an equilibrium distribution. Under ordinary circumstances all matter will have this distribution (including, for example, both the measuring apparatus and what is observed). So generally speaking, the analysis given in the previous section will still hold. However, for a sufficiently fast disturbance, a different distribution may prevail for a short term, and new results may be obtained (for more details see Bohm and Vigier[6]).

These notions were further developed in a somewhat formal way by Nelson[8] and by de la Pena.[9] However, de Broglie used this approach as a physical basis for his thinking, by proposing that, more generally, this fluid is a space-filling background analogous to ether, but with new qualities (it may be regarded, for example, as the physical basis of the "zero point" fluctuations of quantum mechanical field theories). He derived a relativistic extension of the quantum potential of the Klein–Gordon equation from the assumed thermal properties of this medium in a new way in which the mechanical action variable was intrinsically connected to the entropy of the background fluid.

Cufaro Petroni, Droz-Vincent, and Vigier[18] went further along these lines to consider the relativistic many-body system. They proposed that the quantum potential is a reflection of a stochastic process, based on the "zero point" fluctuations. For the special case of a two-body system, they assumed that this potential provides a connection that is instantaneous in the centre of the mass frame of the two bodies. While this violates the spirit of locality which pervades the work of Einstein and de Broglie, they given an argument implying that a consistent account of causality can nevertheless be maintained. We feel however that their argument is not as clear as it might be, while in the manybody system, it appears to be indefinable at what times the particles will be connected, especially considering that in general they all act together inseparably. They are still however working along these lines, and hope eventually to clear up questions of this kind.

We also have been working on this subject, and have proposed a model of a highly nonlinear field with strong vacuum fluctuations (see Bohm and Hiley[19]), which in many ways resembles the idea that de Broglie has suggested, especially in that we take particles as singularities of the field, rather than as *a priori* given entities. However, the main difference is that we focus our attention on the possibility of stable limit cycles for such nonlinear equations. We give arguments showing qualitatively that a completely local set of equations of this kind may give rise, in a suitable limit, to the results of quantum mechanics. This means that, contrary to what is commonly assumed, Bell's inequality may be violated, without implying the need for any nonlocal features of the fundamental field equations.

In addition, one of us (Bohm[16,20]) has developed a model of the relativistic "sub-quantum" background in which de Broglie's notion that each particle has an internal "clock" plays a key part. What is further assumed is a sense of levels, in which each oscillator (that is equivalent to a clock) is a collective coordinate of a set of lower level oscillators to which the same clock relations apply (i.e., these in turn are collective coordinates of a still deeper level). The consistency of the de Broglie relation

$$\oint \sum_i p_1 \, dq_1 = nh$$

for all levels was then demonstrated. This model is able to reproduce many further features of the quantum theory. In particular, by supposing that each level has a zero point motion that is random except for satisfying the clock condition, one is able to deduce the Heisenberg uncertainty relationships directly (so that the appearance of Planck's constant in these relations reflects its presence in the de Broglie condition). This suggestion in principle opens the way for dealing relativistically with the "sub-quantum" medium, explaining all of its quantum properties naturally through de Broglie's clock condition, and connecting this condition with the basic quantum statistical distributions. An essential step that remains to be done is to derive the quantum potential from these assumptions or some extension of them. We hope that some work may be done on this in the future.

Although we are giving considerable attention to the questions discussed in this section, our main work is still as indicated in Section 2, along the lines of development of the implicate order.[16] In particular, we are considering the meaning of attempting to make a quantum theory of the generalized structure of space-time. Wheeler[21] has in fact already been looking at this problem from a topological point of view, in which he starts with his "pre-geometry," and hopes to arrive at ordinary geometry as a limiting case. In our approach, we also have a kind of notion of "pre-geometry," but we start with concepts that are basically a-local, and derive locality as an approximation. This will provide a deeper understanding of those relationships which we have thus far explained through the quantum potential and the de Broglie clock condition.

4. CONCLUSION

First of all, we would like to call attention to the fact that the early work of de Broglie, as reported here, played a key part in making possible the development of the mathematical form of the quantum theory itself. Unfortunately, his physical intuition and imaginative insights were not generally taken up, and therefore did not have a widespread effect on what was subsequently done with the theory. Nevertheless, there is an appreciable number of people who are dissatisfied with such an approach to physics, in which physical intuition has come to play a relatively minor role (perhaps mainly as a convenient aid to working with mathematical formalisms and computations). And indeed some physicists are still working with extensions or developments of de Broglie ideas, or with yet other ideas, which are aimed at a better understanding of the underlying physical reality than is treated by the mathematical formalism alone. What has been done by de Broglie has been much appreciated by these people as an inspiration, and very often as a source of ideas.

APPENDIX[3]

Because some erroneous accounts of how I came to develop the trajectories-interpretation of the quantum theory are in circulation,[22] I thought it advisable to state in some detail how I came to do this work.

To begin with, I wrote a book[23] from Bohr's point of view, mainly in order to try to understand the quantum theory. But after I had written the book, I felt that I still didn't really understand the quantum theory, and so I began to look for new approaches. Meanwhile, I had sent copies of the book to Bohr, Pauli, Einstein, and other scientists. Bohr did not respond, but Pauli sent an enthusiastic reply, saying he liked the book very much. Einstein also got in touch with me, saying that though the book explained the quantum theory about as well as would ever be possible, he still was not convinced, but wanted to discuss the subject with me.

We had several discussions, the net result of which was that I was considerably strengthened in my feeling that there was something fundamental that was missing in quantum theory. This may perhaps have made me work with greater energy, but Y. Ne'eman's statement that I was "shaken" by my conversation with Einstein and "had not recovered to this day" is entirely false. In any case, what actually happened was that I soon came upon the trajectories-interpretation, and prepared a preprint, copies of which were sent to many physicists including de Broglie, Pauli, and Einstein. I learnt shortly thereafter from de Broglie that he had developed this idea much earlier, and so, in later versions of the paper, I adknowledged this fact. Pauli was very negative in his reply, saying also that de Broglie had developed the same model many years earlier, and that it had been shown by him to be wrong at the Solvay Conference.[3]

As a result of Pauli's letter, I developed a theory of the many-body problem answering his objections, which was incorporated in a second paper (Bohm[5]). I had several further discussions with Einstein, but he was not at all enthusiastic about the idea, probably mainly because of the feature of nonlocality of the quantum potential, which conflicted with his basic notion

[3] On the Background of the Papers on Trajectories-Interpretation by D. J. Bohm.

that connections had to be universally local in the fundamental laws of physics.

While I can understand Einstein's objections fully, I feel that it may have been a tactical error on his part to dismiss such ideas because they conflicted with his own notions as to the nature of reality. For though perhaps unsatisfactory in many respects, they made possible, as explained in the present paper, certain important insights into the meaning of the quantum theory. In addition, they helped keep alive an interest in finding a deeper understanding of the quantum theory. I feel that a correct approach might have been to encourage such work as a purely provisional approach, but recognizing that it was not likely in itself to be a fundamental theory, without further radically new ideas. The result of not doing this sort of thing was that, for the most part, fundamental physics was reduced to its present state of relying almost exclusively on formulae and recipes constituting algorithms for the prediction of experimental results, with only the vaguest notions of what these algorithms might mean physically.

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