Homework Five, Physics 242, Spring 2009
Due Wednesday, May 20

[1.] Solve for the eigen-energies of the four site Heisenberg model with near neighbor exhange J and next near neighbor exchange J':

$$H = J(S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_4 + S_4 \cdot S_1) + J'(S_1 \cdot S_3 + S_2 \cdot S_4) \ .$$

You can either construct the 16 state Hilbert space and apply H to each state and diagonalize the resulting matrix, or attempt a trick such as the one we used in class to write H in terms of squares of combined spin operators.

[2.] Reproduce the work we did in class to generate the matrix of the two site Hubbard Hamiltonian in the sector of one spin up and one spin down particle. Then complete the problem by diagonalizing the matrix and verifying the eignevalues stated in class.

<u>Comment:</u> In computing the eigenvalues of a Hamiltonian H, it is often crucial to identify other operators that commute with H. For example, when you solved the Hydrogen atom (or any central potential), the first step was realizing that $[H, L^2] = [H, L_z] = 0$ so that the angular part of the wavefunctions are spherical harmonics. The Hubbard Hamiltonian commutes with the total number of up and down particles: $N_\uparrow = \sum_l n_{l\uparrow}$ and $N_\downarrow = \sum_l n_{l\downarrow}$. Can you argue why this might be from the way creation and destruction operators appear in H? Would it be true for the mean field BCS Hamiltonian? The fact that $[H, N_\uparrow] = [H, N_\downarrow] = 0$ means that H doesn't connect states with different numbers of up and down electrons. H is block diagonal. Use this background to do problem 3.

[3.] The four site Hubbard model has a 256 dimensional Hilbert space. However, the matrix of H is block diagonal. By considering all the possible values of the total number of up and down electrons, compute all the block sizes. Check that they add to 256. Verify the statement in class that the biggest matrix is dimension 36. What is the total dimension of the Hilbert space of the 8 site Hubbard model? What is the largest matrix you would need to diagonalize using the fact that H commutes with $N_\uparrow$ and $N_\downarrow$.

[4.] *Bonus problem for the computationally inclined:* Your computer had 2 Gbytes of memory. How big a vector of double precision (64 bit) numbers can you store in memory? What size Hubbard model would have a vector of this length to describe its wavefunction? You might think that storing H would be even more of a problem. Should you worry about storing H?