Homework Four, Physics 242, Spring 2009

Due Wednesday, May 13

We will do problem zero in class.

[**0.**] Consider the "Periodic Anderson Model" (PAM) written in real space,

$$H = -t \sum_{l\sigma} (c^\dagger_{l+1,\sigma} c_{l\sigma} + c^\dagger_{l\sigma} c_{l+1,\sigma}) + V \sum_{l\sigma} (c^\dagger_{l\sigma} d_{l\sigma} + d^\dagger_{l\sigma} c_{l\sigma}) + \epsilon_d \sum_{l\sigma} d^\dagger_{l\sigma} d_{l\sigma} \ . \qquad (1)$$

This Hamiltonian describes a set of delocalized (conduction) electrons, represented by the "c" operators, which have a hopping (kinetic energy) from site to site, hybridized to a set of localized electrons (no intersite hopping), represented by the "d" operators. By going to k space and doing an additional rotation, compute the energy levels (energy bands) of the PAM.
Note: The PAM is very commonly used to describe heavy fermion materials, when an interaction term $U \sum_l d^\dagger_{l\uparrow} d_{l\uparrow} d^\dagger_{l\downarrow} d_{l\downarrow}$ is added to H.

[**1.**] Compute the Green's function,

$$G_\uparrow(k,t) = -i \, \langle T \, c_{k\uparrow}(t) c^\dagger_{k\uparrow}(0) \, \rangle \ , \qquad (2)$$

of the Periodic Anderson model (written here in momentum space),

$$H = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + V \sum_{k\sigma} (c^\dagger_{k\sigma} d_{k\sigma} + d^\dagger_{k\sigma} c_{k\sigma}) + \epsilon_d \sum_{k\sigma} d^\dagger_{k\sigma} d_{k\sigma} \ . \qquad (3)$$

Use the same "equation of motion" technique as in class for the BCS Hamiltonian. In close analogy with our Green's function solution for BCS, you will find that $G_\uparrow(k,t)$ couples to,

$$F_\uparrow(k,t) = -i \, \langle \, T d_{k\uparrow}(t) c^\dagger_{k\uparrow}(0) \, \rangle \ , \qquad (4)$$

Writing the equation of motion for F produces a closed set of equations for G and F which you can solve in frequency space. What is the "self-energy" of a conduction electron in the PAM? What happens if you tried this approach with the U term mentioned in problem zero present?

[**2.**] Read Leggett's Nature Physics paper "What DO we know about High-T_c?," and answer **two** of the following questions. A one paragraph response is fine.

1. What experimental evidence is there for Leggett's second claim that "The principal locus of superconductivity in the copper oxygen planes"?
2. Leggett's third claim is that the different copper oxygen planes are independent. Is there any literature on a correlation between the number of closely spaced copper oxygen planes and the superconducting transition temperature? Does this invalidate Leggett's assertion?
3. How does the NMR Knight shift and spin-lattice relaxation rate tell you the spin state of a Cooper pair (Leggett's fifth claim).
4. Leggett's sixth claim mentions "tetragonal symmetry". What is he talking about?
5. How do measurements of the upper critical field of a superconductor tell you the size of the pairs (Leggett's seventh point).
6. What is the BEC-BCS crossover mentioned in Leggett's seventh point?