Homework Two, Physics 242, Spring 2009
Due Monday, April 20

[**1.**] If you insert "appropriate" numbers into the London formula for the penetration length, $\lambda_L^2 = m/(ne^2\mu_0)$, what sorts of lengths do you get? Assume all the conduction electron density n is available to form the superfluid, and pick values for n of a typical metal. The different materials you looked at in Problem Set 1 have different values of n. Is there any rough consistency between the λ_L values you obtain for these different n and the experimental values for the penetration length?

[**2.**] In class we discussed the fact that the suppression of $C(T)$ at low T from a linear to exponential behavior which occurs as a result of the superconducting transition should be accompanied by an enhancement of $C(T)$ for some other T range in order to keep $S(\infty) = \int_0^\infty dT\, C(T)/T$ fixed. Sketch $C(T)$ for the two level system $E = 0, \Delta$ of Problem Set 1 for $\Delta = 3$. Where is the peak? Why is it there? Suppose something happens to the system (the analog of a superconducting transition) which shifts Δ from $\Delta = 3$ to $\Delta = 7$. What happens to $C(T)$? Is the suppression of a peak for one range of T accompanied by a reappearance elsewhere? How does $\int_0^\infty dT\, C(T)/T$ differ for the two values $\Delta = 3$ and $\Delta = 7$?

[**3.**] In class we saw that the tight binding Hamiltonian describing hopping of electrons between near neighbor sites on a one dimensional lattice in real space,

$$H = -t \sum_l (c_{l+1}^\dagger c_l + c_l^\dagger c_{l+1}),$$

could be transformed to momentum space,

$$H = \sum_k \epsilon(k)\, c_k^\dagger c_k,$$

where the "energy band" $\epsilon(k) = -2t\cos(k)$.

Do the same transcription to momentum space for a two dimensional square lattice with near neighbor hopping. What is $\epsilon(k_x, k_y)$? What happens if there is an additional next near neighbor hopping t' across the diagonal of a square? (You can begin to see how the band structure of a material might be fit by assigning appropriate hoppings of various ranges in a tight binding model.)

[**4.**] Draw the Fermi surface (the curve of constant energy E) for the two dimensional square lattice of the preceding problem. Sketch the result for $E = -3t, E = -2t, E = -t$, and $E = 0$. We will discuss the significance of the $E = 0$ case in class. This model, and the special properties of its Fermi surface, are fundamental to many theories of high temperature superconductors, as we shall discuss in several weeks.