Physics 241– Quantum Magnetism

Problem Set 5 Due 2/27/03

[1.] Write a program to diagonalize the two site Hubbard model. Take the ground state eigenfunction in the sector with one up and one down electron and evaluate the 'local moment' $\langle (n_\uparrow - n_\downarrow)^2 \rangle$ as a function of U/t. Plot it.

[2.] (This problem is non-trivial. Please feel free to email me about it.) Complete the solution of the Hubbard model in Mean Field Theory that we began in class by actually evaluating (numerically) the energy and seeing for which *ansatz*, paramagnetic, ferromagnetic, or antiferromagnetic, it is minimized. Take some fairly large lattice in one dimension and construct the phase diagram as a function of U/t and filling (number of electrons per site). By this I mean, figure out the regions where each phase, paramagnetic, ferromagnetic, or antiferromagnetic, has lowest energy.

[3.] Consider a model of 'spinless fermions' in one dimension:

$$H = -t \sum_l (c^\dagger_{l+1} c_l + c^\dagger_l c_{l+1}) + V \sum_l n_l n_{l+1}$$

Note that the fermions have no spin index σ: there is only one type of fermion in this problem. Since there is only one spin type, there can be no on-site interaction, so we have included an intersite repulsion V. By thinking about the action of H on the states in the Hilbert space, argue that this model is that same as the Heisenberg model if V takes on a special value relative to t. What is that value? The key point is that the minus signs which one gets for the fermion hopping don't arise in 1-d (for near neighbor hopping.) This is one of many examples of mappings between boson, fermion, and spin models which occur in one dimension (and *only* in one dimension). There is one subtlety: What happens with the boundary conditions?! When a fermion hops from the last site to the first, a minus sign can occur which doesn't occur in the Heisenberg model. Under what conditions of filling (number of fermions on the lattice) does this happen? How can you save the mapping? Hint: What about anti-periodic boundary conditions?