PROBLEM SET 4 Due Tuesday November 19

Physics 240A– FALL 2019

[**1.**] What is the relation between the electron density n and the Fermi wavevector k_f in **two dimensions**? What does the density of states $N(E)$ look like for the usual quadratic dispersion relation?

[**2.**] Draw the Fermi surfaces (trajectories in the k_x, k_y plane with constant energy E) for

$$E(k) = -2t\,[\,\cos(k_x) + \cos(k_y)\,].$$

(This is the famous simplified dispersion relation for electrons in the cuprate superconductors.) First analyze the cases when (a) only a small fraction of the k points are occupied (low density); (b) $E = 0$; and (c) when almost all the k points are occupied. You should be able to do this without resorting to a computer. In case (b) what fraction of the k-points are occupied? Draw the Fermi surfaces for $E = -3.5\,t, -3.0\,t, -2.5\,t, -2.0\,t, -1.5\,t, -1.0\,t, -0.5\,t$. Here a computer program is useful!

[**3.**] Write an expression for the density of states $N(E)$ for the dispersion relation of problem 2. See how far you can get in evaluating it analytically. This turns out to be non-trivial (elliptic integrals) although some approximate things can be said more simply. One thing to try to show is that $N(E)$ diverges at $E = 0$. A much less demanding approach is just to write a small program similar to that discussed in class for $E(k) = -2t\cos(k)$.

[**4.**] Find the pressure of the free electron gas in three dimensions and zero temperature. Start from $P = -\partial E/\partial V$, in analogy to what we did in class for the ideal gas law at finite T.

[**5.**] The crucial ideas of the Sommerfeld expansion were discussed in class. Work through the derivation (e.g. in Achcroft and Mermin.) Nothing to hand in here $\cdots$.