1. Kinetic theory of gases : molecules are identical solid spheres which move in straight lines until they collide with each other. collision time is very fast, and forces between molecules act only in collisions.
Drude's idea: Treat e^- in metal the same way

~~Basic assumptions of Drude model~~ :

Why might you worry about this idea. The electrons are much closer together than gas molecules. Estimate this number

~~[illegible]~~ ~~[illegible]~~ $6.022 \cdot 10^{23}$ $\frac{\text{atoms}}{\text{mole}}$

copper

density = $8.9\ g/cm^3$

1 mole = 63.5 g

$\Rightarrow$ $0.844 \cdot 10^{23}$ atoms$/cm^3$

Valence $Z = 1$

$\Rightarrow$ $0.844 \cdot 10^{23}\ e^-/cm^3$ $\leftarrow$ typical separation is cube root of this (number)$^{-1}$
$\sim 10^{-8}$ cm
$\sim$ 1 Bohr radius.
$0.529 \cdot 10^{-8}$ cm

Gas molecules density is 10^3 times less ie volume occupied is 1000 x bigger.

Basic Assumptions

(1) Between collisions, electrons travel in straight line with constant $\vec{v}$, unless external field applied in which case the evolution of $\vec{v}$ given by Newton's eqns i.e. neglect el-el interaction & el-ion interactions

(2) except as providing collisions which abruptly alter e^- velocity, & immediately relax it to appropriate temp T.

(3) e^- travel on average time τ between collisions

~~[illegible]~~

Implications of Drude Theory

(1) DC conductivity

$$\vec{j} = -ne\vec{v}$$

Vdt

$$dq = \frac{\#\,el}{Volume} \cdot volume\ \frac{q}{el} = nAVdt\,e$$

$$\langle V \rangle = -\frac{eE\tau}{m}$$

$$j = \frac{1}{A}\frac{dq}{dt} = neV$$

$$j = \underbrace{\frac{ne^2\tau}{m}}_{\sigma} E$$

From our knowledge of n, e, m, $\rho \simeq 1\ \mu\Omega\text{-cm} = 10^{-8}\ \Omega\text{-m}$

$\sigma = 1/\rho = 10^8$

can get $\tau \sim 10^{-14}$ sec

$$\cancel{\tau = \frac{m\sigma}{ne^2} \sim \frac{9\cdot 10^{-28}\ 10^{-6}}{10^{23}\ (4.8\cdot 10^{-10})^2} \sim 10^{-37}}$$

$$\tau = \frac{m\sigma}{ne^2} \sim \frac{9\cdot 10^{-31}\ 10^{+8}}{10^{29}\ (1.6\cdot 10^{-19})^2} = \frac{9}{2.6}\ 10^{-14}$$

AC conductivity

$$p(t+dt) = \left(1 - \frac{dt}{\tau}\right)\left[p(t) + f(t)\,dt\right]$$

$$= p(t) - \frac{dt}{\tau} p(t) + f(t)\,dt$$

$$dp/dt = -p(t)/\tau + f(t)$$

What happens if $f(t) = 0$?

$$f(t) = eE(t) = \mathrm{Re}\left(eE_0(\omega) e^{-i\omega t}\right)$$

$$p(t) = \mathrm{Re}\left[p_0(\omega) e^{-i\omega t}\right]$$

$$-i\omega p(\omega) = -p(\omega)/\tau + -eE(\omega) \Rightarrow p(\omega) = \frac{-eE(\omega)}{1/\tau - i\omega}$$

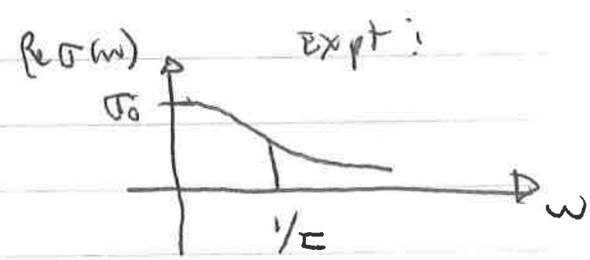
$$j(\omega) = -\frac{ne}{m} p(\omega) = \frac{+\,ne^2/m\, E(\omega)}{1/\tau - i\omega}$$

$$j(\omega) = \frac{ne^2\tau/m\, E(\omega)}{1 - i\omega\tau}$$

$\omega = 0$ limit recover dc result.

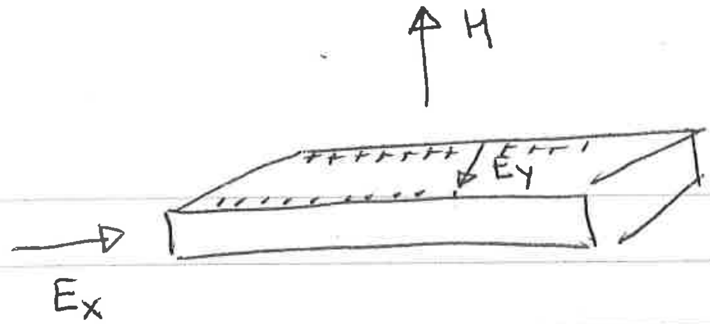
$$\sigma(\omega) = \sigma_0/1 - i\omega\tau \qquad \sigma_0 = ne^2\tau/m$$

$$\mathrm{Re}\,\sigma(\omega) = \sigma_0/1+\omega^2\tau^2$$



4.

(3) Hall Effect



over

E_x leads to current j_x

$\Rightarrow$ ~~for~~ H leads to Lorentz force on e^- so electrons pile up and yield E_y

~~Hall coefficient $R_0 =$~~

$$\frac{dp}{dt} = -p/\tau - e(\vec{E} + \vec{p}/mc \times \vec{H})$$

$p_x \quad p_y \quad p_z$

$0 \quad 0 \quad H$

steady state $dp/dt = 0$

$\omega_c = eH/mc$

cyclotron freq

$$0 = -p_x/\tau - e(E_x - \omega_c p_y)$$

$$0 = -p_y/\tau - e(E_y + \omega_c p_x)$$

$\frac{mv^2}{r} = evB$

$m\omega^2 r = e\omega r B$

$\omega = \frac{eB}{m}$

Multiply by $-ne\tau/m$ $\quad j = -nep/m$

$$\sigma_0 E_x = \omega_c \tau j_y + j_x$$

$$\sigma_0 E_y = -\omega_c \tau j_x + j_y$$

E_y is given by requiring $j_y = 0$

$$\sigma_0 E_x = j_x$$

$$\sigma_0 E_y = -\omega_c \tau j_x$$

$\leftarrow$ <u>continuous</u> Hall. H does not enter relation between j_x and E_x.

$$\Rightarrow E_y = -\frac{\omega_c \tau}{\sigma_0} j_x = -\frac{H}{nec} j_x$$

$$R_H = -1/nec$$

R_H depends only on n not τ. if Drude correct

metal	$-1/R_H nec$
Li	0.8
Na	1.2
K	1.1
Rb	1.0
Cs	0.9
Cu	1.5
Ag	1.3
Au	1.5.

(4) Dielectric constant of a metal.

$$\nabla\cdot E = 0$$

$$\nabla\cdot H = 0$$

$$\nabla\times E = -\frac{1}{c}\,\partial H/\partial t$$

$$\nabla\times H = \frac{4\pi}{c} J + \frac{1}{c}\,\partial E/\partial t$$

Assume time dependence $e^{-i\omega t}$

$$\nabla\times(\nabla\times E) = -\nabla^2 E + \nabla(\cancel{\nabla\cdot E})^{\,0}$$

$$-\nabla^2 E = \frac{i\omega}{c}\nabla\times H = \frac{i\omega}{c}\left(\frac{4\pi}{c}\sigma E - \frac{i\omega}{c}E\right)$$

$$= \frac{\omega^2}{c^2}\left[1 + \frac{4\pi i\sigma}{\omega}\right]E$$

This is the wave eqn $\quad -\nabla^2 E = \frac{\omega^2}{c^2}\epsilon E$

with $\quad \epsilon(\omega) = 1 + \frac{4\pi i\sigma}{\omega}$

6.

$$\epsilon(\omega) = 1 + \frac{4\pi i}{\omega}\left[\frac{\sigma_0}{1-i\omega\tau}\right]$$

Suppose $\omega\tau >> 1$ [$\omega >> 10^{14}$ sec^{-1}

visible $400-700 \cdot 10^{-9}$ m

$\nu\lambda = c$

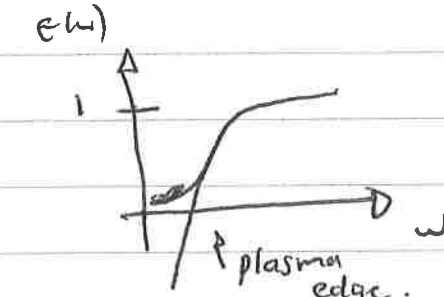
$\omega = 2\pi c/\lambda = \frac{6 \cdot 3 \cdot 10^8}{5 \cdot 10^{-7}} = 4 \cdot 10^{15}$]

$$\epsilon(\omega) = 1 - \frac{4\pi\sigma_0}{\omega^2\tau}$$

$$\sigma_0 = ne^2\tau/m$$

$$= 1 - \frac{4\pi ne^2/m}{\omega^2}$$

$$\epsilon(\omega) \equiv 1 - \omega_p^2/\omega^2$$



What happens if $\epsilon(\omega) < 0$?

$\omega < \omega_p$ reflects

material reflects $E(\omega)$ decays exponentially in metal

$\omega_p^2 = \frac{4\pi ne^2}{m}$

$\frac{10 \cdot 10^{29}(10^{-19})^2}{10^{-30}} \approx 10^{22}$

$\sim 10^{11}$

	Theory $\lambda(10^3 \text{ Å})$	observed λ
Li	1.5	2.0
Na	2.0	2.1
K	2.8	3.1
Rb	3.1	3.6
Cs	3.5	4.4

Analogy: Reflection of radio waves by ionosphere

$\omega_p \sim n$ is much smaller

M sug

$\tau \approx 1000$

$\omega_p \approx 10^3/\tau \approx 10^{17}$

0.

Maxwell Eqns

$\vec{\nabla}\cdot\vec{D} = 4\pi\rho$ $\quad$ $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$ $\quad$ $\vec{D} = \vec{E} + 4\pi\vec{P}$

$\vec{H} = \vec{B} - 4\pi\vec{M}$

$\vec{\nabla}\cdot\vec{B} = 0$ $\quad$ $\vec{\nabla}\cdot\vec{B} = 0$ $\quad$ + "constitutive relns" between $\vec{E},\vec{B}$ to $\vec{D},\vec{H}$

$\vec{\nabla}\times\vec{H} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\,\partial\vec{D}/\partial t$ $\quad$ $\vec{\nabla}\times\vec{B} = \mu_0 J + \partial\vec{E}/\partial t$ $\quad$ $\vec{D} = \epsilon\vec{E}$

$\vec{\nabla}\times\vec{E} + \frac{1}{c}\,\partial\vec{B}/\partial t = 0$ $\quad$ $\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$ $\quad$ $\vec{H} = \mu\vec{B}$

some materials are "linear" and "isotropic"

very complicated in general to relate $\vec{P},\vec{M}$ — really depend on details of material

~~Why $\vec{D}$ instead of E~~

no spins → Diamagnetic $\quad \mu - 1 \sim 10^{-4}$

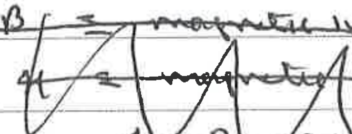
~~e~~ field induces extra current loops opposing original field

Paramagnetic $\quad 1-\mu \sim 10^{-2}$ to 10^{-5}

unpaired electron → free spins

$\vec{m}$ aligns with field and

Ferromagnets all line up creates dom



~~B = magnetic induction~~

~~H = magnetic field~~

~~H > B paramagnetic~~

~~→ $\mu > 1$~~

If we set $\vec{H} = \vec{B}$

∴ 2 questions

~~v~~ $\quad \vec{\nabla}\cdot\vec{E} = 0$ vs $\vec{\nabla}\cdot\vec{D} = 4\pi\rho$

$\nabla\times\vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$ vs $\nabla\times H = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{D}}{\partial t}$

why free $\vec{J}$ but no $\vec{P}$?!

1.

Better motivation of Drude Model

1860 - 1890 kinetic theory of gases

1897 Thomson discovers e^- - a natural candidate for the particle to carry current

1900 Drude begins treatment of e^- analogous to gas molecules

(0) Error in $dp/dt = -p/\tau + f$ $\longrightarrow$ over

(1) ~~Free~~ motion between scattering is governed by classical eq of motion $F = m\dot{v}$, F only due to external E, B fields, as opposed to interaction

(2) All details of scattering in single number τ, the time between collisions and $\langle \vec{v} \rangle = 0$; $\frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}kT$; after collision

~~to~~ No theory for τ!

$\therefore$ useful only if τ indep quantities available

(i) Hall Effect $R_H = -1/nec$ only depends on carrier density. Agrees with expt.

$\frac{E_y}{j_x H}$ $j_x = \sigma E_x$ Hall had observed the fact that σ didn't depend on $\vec{H}$.

2.

(2) Second quantity comes from thermal conductivity

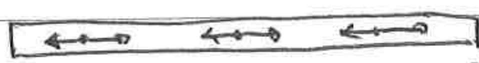
Fourier's law $\vec{j}^{\,q} = K\vec{\nabla}T$

Weidemann-Franz (1853) observe $\quad K/\sigma T = \text{constant}$

1-d calculation

$j = nve \qquad\qquad j^q = nv\mathcal{E}$

particles/volume, velocity, q/particle $\qquad$ Energy/particle



T high $\qquad\qquad$ T low

Just as many move left/right
but ones moving right carry more $\mathcal{E}$ than left
→ Energy gradually transferred to ~~left~~ right.

Energy per particle $\quad \mathcal{E} = \tfrac{3}{2}kT = \tfrac{1}{2}m\langle v^2\rangle$

T is a function of x.

x

$$j^q = nv\left[\mathcal{E}(T(x - v\tau)) - \mathcal{E}(T(x+v\tau))\right]$$

electrons moving to right had last collision at $x - v\tau$

to left had last collision at $x + v\tau$

Why only $\mathcal{E}(x)$? Why not $\vec{v}$? ~~[illegible]~~

Answer: If v were really different a net electric current would also flow. As in Hall effect a small charge builds up at ends of sample to counter this effect.

$$j^q = nv \frac{d\mathcal{E}}{dT}\frac{dT}{dx}(-2v\tau)$$

$$= \underbrace{nv^2\tau \frac{d\mathcal{E}}{dT}}_{\kappa}\left(-\frac{dT}{dx}\right)$$

In 3-d

$$\kappa = \tfrac{1}{3} n v^2 \tau \, d\mathcal{E}/dT \qquad \mathcal{E} = \tfrac{3}{2}kT$$

$$d\mathcal{E}/dT = \tfrac{3}{2}k$$

$$= \tfrac{1}{3} n \left[3kT/m\right] \tau \, \tfrac{3}{2}k$$

$$= \tfrac{3}{2} n (k^2\tau/m) T \qquad \sigma = ne^2\tau/m$$

$$= \tfrac{3}{2}(k/e)^2 T \sigma$$

$$k/\sigma T = \tfrac{3}{2}(k/e)^2 \qquad \text{"Lorentz number"}$$

$$= 1.11 \cdot 10^{-8} \ \frac{\text{watt}-\Omega}{K^2}$$

Wholly accidental agreement. Expressions for κ, σ each off by ×100. $\mathcal{E} = 3/2\,kT$ is completely wrong!

Table 1.6
EXPERIMENTAL THERMAL CONDUCTIVITIES AND LORENZ NUMBERS OF SELECTED METALS

	273 K		373 K	
ELEMENT	κ (watt/cm-K)	$\kappa/\sigma T$ (watt-ohm/K^2)	κ (watt/cm-K)	$\kappa/\sigma T$ (watt-ohm/K^2)
Li	0.71	2.22×10^{-8}	0.73	2.43×10^{-8}
Na	1.38	2.12		
K	1.0	2.23		
Rb	0.6	2.42		
Cu	3.85	2.20	3.82	2.29
Ag	4.18	2.31	4.17	2.38
Au	3.1	2.32	3.1	2.36
Be	2.3	2.36	1.7	2.42
Mg	1.5	2.14	1.5	2.25
Nb	0.52	2.90	0.54	2.78
Fe	0.80	2.61	0.73	2.88
Zn	1.13	2.28	1.1	2.30
Cd	1.0	2.49	1.0	
Al	2.38	2.14	2.30	2.19
In	0.88	2.58	0.80	2.60
Tl	0.5	2.75	0.45	2.75
Sn	0.64	2.48	0.60	2.54
Pb	0.38	2.64	0.35	2.53
Bi	0.09	3.53	0.08	3.35
Sb	0.18	2.57	0.17	2.69

Source: G. W. C. Kaye and T. H. Laby, *Table of Physical and Chemical Constants*, Longmans Green, London, 1966.

As a concrete example let us examine a case where the temperature drop is uniform in the positive x-direction. In the steady state the thermal current will also flow in the x-direction and have a magnitude $j^q = -\kappa\, dT/dx$. To calculate the thermal current we note (assumption 4, page 6) that after each collision an electron emerges with a speed appropriate to the local temperature; the hotter the place of the collision, the more energetic the emerging electron. Consequently, even though the mean electronic velocity at a point may vanish (in contrast to the case when an electric current flows) electrons arriving at the point from the high-temperature side will have higher energies than those arriving from the low-temperature side leading to a net flow of thermal energy toward the low-temperature side (Figure 1.6).

To extract a quantitative estimate of the thermal conductivity from this picture, consider first an oversimplified "one-dimensional" model, in which the electrons can only move along the x-axis, so that at a point x half the electrons come from the high-temperature side of x, and half from the low. If $\mathcal{E}(T)$ is the thermal energy per electron in a metal in equilibrium at temperature T, then an electron whose last collision was at x' will, on the average, have a thermal energy $\mathcal{E}(T[x'])$. The electrons arriving at x from the high-temperature side will, on the average, have had their last collision at

9.

$$\omega_p \tau \simeq 10^3$$

$$\omega_p \simeq 10^{17}$$

~~If you are~~

(3) Return to dielectric constant.

$$\epsilon(\omega) = 1 + \frac{4\pi i}{\omega}\sigma$$

$$= 1 + \frac{4\pi i}{\omega}\left(\frac{\sigma_0}{1 - i\omega\tau}\right)$$

suppose $\omega \gg \frac{1}{\tau} \sim 10^{14}$ light frequencies

$\sim 10^{15}$ $\sim 4 \cdot 10^{15}$

$$\epsilon(\omega) = 1 - \frac{4\pi\sigma_0/\tau}{\omega^2} = \qquad \sigma_0 = ne^2\tau/m$$

$$= 1 - \omega_p^2/\omega^2 \qquad \omega_p \equiv 4\pi ne^2/m \approx 10^{17}\,\text{sec}^{-1}$$

when $\epsilon(\omega) < 0$ waves reflect (decay exponentially in the medium)

So predict for ~~large small ω metals~~ ω

$\omega \lesssim \omega_p$ metals reflect, then become transparent when $\omega \gtrsim \omega_p$.

Indeed metals do become transparent in ultraviolet.

5.

(1) Some simplifying assumptions about collisions and motion between collisions (role of ions)

+ (2) some assumptions about distribution of velocities.

Summary: ~~I will come back to~~ Drude model has some apparent success → Hall is basically correct

Dielectric constant makes sense; Wiedemann Franz law apparently okay

But really this is very wrong.

$\varepsilon = 3/2\, kT$

$\Rightarrow c = 3/2\, k$ per particle. Never observed ~~much~~ much smaller

This is a consequence of using Boltzmann distribution

$$P(E) = e^{-\beta E} = e^{-\beta \frac{1}{2} m v^2}$$

and we can easily fix it up.

$$f(E) = \frac{1}{e^{\beta E} + 1}$$