[1.] Show that the projection operator $T_b(S'|S_1, S_2, S_3)$ for the majority rule for the three site cell discussed in class can be written explicitly in the form

$$T_b = \frac{1}{2} \left( 1 + S' \left( \lambda (S_1 + S_2 + S_3) + \mu S_1 S_2 S_3 \right) \right)$$

for some values of $\lambda, \mu$. Use this expression to calculate the same RG equations as we derived in class, for the two cell approximation with $B = 0$. It helps if you write

$$e^{JS_1 S_2} = \cosh \bar{J} \left( 1 + S_1 S_2 \tanh \bar{J} \right)$$

and use the same ideas as in the high $T$ expansion.

[2.] Any $T_b$ of the form above should give a RG transformation, since $\sum_{S'} T_b = 1$. In any approximation scheme, the results will depend on $\mu$ and $\lambda$. Take the case $\mu = 0$ and calculate how the fixed point $\bar{J}_*$ depends on $\lambda$. If you adjust $\lambda$ so that $\bar{J}_*$ is at its exact value $0.275 \ldots$, how well do you do on the critical exponent $\nu$?