PROBLEM SET 4
Physics 219A, Spring 2014
Due Wednesday, May 14

[1.] Work out the mean field theory for the Blume-Capel model

\[ H = -J \sum_{ij} S_i S_j + \Delta \sum_i S_i^2 \]

where \( S_i = 0, \pm 1 \). Show that for a range of values of \( \Delta \) the model has a first order phase transition, that is, the magnetization \( M \) is a discontinuous function of the temperature \( T \). Sketch the phase diagram. Hints:

i. Write \( S_i = M + (S_i - M) \) and substitute into \( H \) neglecting terms of order \((S - M)^2\).

ii. Calculate \( Z \) and \( F \) as a function of \( M \).

iii. Sketch graphs of \( F \) versus \( M \) for various \( \Delta \) and \( T \). A crucial feature compared to the models we discussed in class will be the possibility that the \( M^4 \) term in \( F \) could be negative.

[2.] Solve the XY model in mean field theory. What is \( T_c \) for a 2-d square lattice, and how does it compare to the mean field solution for the Ising model? Can you argue why \( T_c \) is expected to be lower/higher?