[1.] Construct the high temperature expansion for the partition function $Z$ of the 2-d Ising model on a triangular lattice. Go out to at least sixth order in $t = \tanh(\beta J)$. Evaluate the free energy $F$ and show that all terms of higher power than linear in $N$ cancel out, so that $F$ is properly extensive.

[2.] Construct the high temperature expansion for the nearest neighbor spin-spin correlation function of the 2-d Ising model on a triangular lattice.

[3.] Construct the low temperature expansion for the partition function $Z$ of the 2-d Ising model on a triangular lattice. Do the high and low temperature expansions look related, as is the case for a square lattice? Actually, the low $T$ expansion of the Ising model on a triangular lattice is related to the high $T$ expansion on a different lattice. What is that lattice?

[4.] Look up the Onsager solution of the 2-d Ising model on a square lattice, and plot the energy $E$ as a function of temperature $T$. On the same graph, plot the high and low temperature expansion results for $E(T)$.

[5.] Solve the 1-d “XY” model by the transfer matrix technique. The Hamiltonian is

$$H = -J \sum_l \cos(\theta_l - \theta_{l+1}).$$

On each site $l$ we have an angular variable $\theta_l$ which can take on any value $0 \leq \theta_l \leq 2\pi$. Unlike the Ising case, where the transfer matrix $M$ is finite dimensional, in the XY model $M$ is infinite dimensional. (But you have encountered such things already in quantum mechanics.) The eigenvalues are the solution of an appropriate integral equation.

Hint:

$$\exp(\beta J \cos(\theta - \theta')) = \sum_n I_n(\beta J) \exp(in(\theta - \theta'))$$

where $I_n(x)$ are Bessel functions. (Apparently they are everywhere!)