PROBLEM SET 3

Physics 219A, Spring 2014

Due Wednesday, April 30

[**1.**] Construct the high temperature expansion for the partition function Z of the 2-d Ising model on a *triangular* lattice. Go out to at least sixth order in $t = \tanh(\beta J)$. Evaluate the free energy F and show that all terms of higher power than linear in N cancel out, so that F is properly extensive.

[**2.**] Construct the high temperature expansion for the nearest neighbor spin-spin correlation function of the 2-d Ising model on a triangular lattice.

[**3.**] Construct the low temperature expansion for the partition function Z of the 2-d Ising model on a *triangular* lattice. Do the high and low temperature expansions look related, as is the case for a square lattice? Actually, the low T expansion of the Ising model on a triangular lattice is related to the high T expansion on a *different* lattice. What is that lattice?

[**4.**] Look up the Onsager solution of the 2-d Ising model on a square lattice, and plot the energy E as a function of temperature T. On the same graph, plot the high and low temperature expansion results for $E(T)$.

[**5.**] Solve the 1-d "XY" model by the transfer matrix technique. The Hamiltonian is

$$H = -J \sum_l \cos(\theta_l - \theta_{l+1}).$$

On each site l we have an angular variable θ_l which can take on any value $0 \leq \theta_l \leq 2\pi$. Unlike the Ising case, where the transfer matrix M is finite dimensional, in the XY model M is infinite dimensional. (But you have encountered such things already in quantum mechanics.) The eigenvalues are the solution of an appropriate integral equation.
<u>Hint:</u>

$$\exp(\,\beta J \cos(\theta - \theta')\,) = \sum_n I_n(\beta J) \exp(\,in(\theta - \theta')\,)$$

where $I_n(x)$ are Bessel functions. (Apparently they are everywhere!)