

PROBLEM SET 2

Physics 219A, Spring 2014

Due Wednesday, April 16

[1.] In class we computed the density of states $N(E)$ for N free particles,

$$H = \sum_{l=1}^N \frac{p_l^2}{2m}, \quad (1)$$

directly from the definition. Do the calculation by a second method:

a. First, show

$$Z(\beta) = \int d\Gamma e^{-\beta H} \quad (2)$$

is the Laplace transform of $N(E)$.

b. Compute $Z(\beta)$ by combining Eqns. 1 and 2 and doing the resulting (trivial) Gaussian integrals.

c. Compute $N(E)$ using your expression for $Z(\beta)$ from part (b) and the usual techniques for computing an inverse Laplace transform. You should get the same result we got in class.

[2.] The ideal relativistic gas. This problem can be done by following the computation done in class for the non-relativistic case where the energy-momentum dispersion relation is quadratic. In fact, the algebra is slightly simpler. You might want to get $N(E)$ using the approach of problem (1).

a. Calculate $N(E) = \int d\Gamma \delta(E - H)$ for N free classical particles in the extreme relativistic limit,

$$H = \sum_l cp_l. \quad (3)$$

b. Find the equation of state and the specific heat.

c. Find the single particle momentum distribution function.

[3.] Consider a single simple harmonic oscillator,

$$H = \frac{1}{2}m\omega^2 x^2 + \frac{1}{2m}p^2 \quad (4)$$

in the microcanonical ensemble.

a. Draw a “phase space” picture of the trajectory of the particle assuming it has energy E . Can you predict what the probability distributions $P(x)$ and $P(p)$ of position and momentum might look like from this picture?

b. Compute the density of states $N(E)$.

c. Compute $P(x)$ and $P(p)$. Does your answer agree with your prediction from (a)?

d. In the canonical ensemble, as we have discussed in class, we imagine the oscillator in contact with a heat bath at temperature T rather than having some fixed energy E . What are $P(p)$ and $P(x)$ in that case? Where are they peaked? How do they compare with the microcanonical result of (c)?

e. Should we fret about the difference? After all, the different ensembles are supposed to be equivalent!

[4.] Consider N independent particles each of which can assume two energy levels 0 and ϵ . Denote by n_0 the number of particles in the energy level 0 and by n_1 the number in level ϵ .

a. What is the relation between the energy E and n_1 ?

b. What is the relation between the entropy S and n_1 ?

c. Using $1/T = dS/dE$, compute $E(T)$. Make a plot of the result for $N = 128$ particles. For the plot, choose $k_B = \epsilon = 1$.

d. On the same plot, show the canonical result

$$\langle E \rangle (T) = \frac{N}{e^{1/T} + 1}. \quad (5)$$

What do you conclude?

e. Comment on what happens to the temperature for energies $E > N\epsilon/2$. Do you know a physical system which has this sort of behavior?