PROBLEM SET 1

Physics 219A, Spring 2014

Due Monday, April 7

[**1.**] Consider a classical system which has three energy states $E_1 < E_2 < E_3$.
a. What is the partition function Z?
b. What is the free energy F?
c. What are the probabilities p_1, p_2, p_3 of occupying each state? Sketch $p_i(T)$ versus T and interpret.
d. Show $\langle E \rangle = \partial \ln Z / \partial \beta$ gives the same answer as $\langle E \rangle = \sum_i p_i E_i$. Sketch $\langle E \rangle(T)$ versus T.
e. Compute $C = d\langle E \rangle / dT$ and show it equals $\beta^2 (\langle E^2 \rangle - \langle E \rangle^2)$.
f. What is the entropy $S(T)$? What are its low and high temperature limits?

[**2.**] In this problem we'll consider the limiting behavior of the specific heat at low and high temperature and what it might tell us about the system's energy states.
a. The specific heat of the two state system we considered in class obeys $C(T \to \infty) \to 0$. On the other hand, for a classical harmonic oscillator $C(T) = k_B$ for all T, and hence does not vanish as $T \to \infty$. What can you say about the energy states of a system which would affect whether or not C vanishes at high T?
b. The specific heat of the two state system we considered in class obeys $C(T \to 0) \to 0$, while that of the harmonic oscillator remains finite. What can you say about the energy states of a system which would affect whether or not C vanishes at low T?
c. Some glasses have a specific heat which goes to zero *linearly* with T as $T \to 0$. A model for this phenomenon is that inside the glass are *many* two level systems (corresponding, perhaps, to two different positions of the atoms). Assume that these two level systems can have a distribution of energy spacings Δ and that the number of two level systems with spacing Δ is the same for all Δ. Show that this results in $C(T)$ which is linear in T.
d. The specific heat in (c) goes to zero as $T \to 0$, but does so more slowly (linearly rather than exponentially) that a single two level system of spacing Δ_0. Explain why physically. Make a general statement about what you might infer from the knowledge that $C(T)$ vanishes exponentially. (This was an important clue into the mystery of superconductivity.)

[**3.**] Suppose a system has an infinite number of states with discrete energies $E_n = n\Delta$ with $n > 0$. Compute the partition function Z, the free energy $F(T)$, the average energy $\langle E \rangle(T)$, and the entropy $S(T)$.

[**4.**] We will use the results of this exercise a fair amount in this course: Given a binomial distribution $\mathcal{P}(n) = \binom{N}{n} p^n q^{N-n}$. Evaluate $\langle n \rangle = \sum_n n \, \mathcal{P}(n)$. Hint: Consider the expression for $\sum_n \mathcal{P}(n)$ and differentiate with respect to p. Compute also $\langle n^2 \rangle$ and then $\langle n^2 \rangle - \langle n \rangle^2$. Is there any physical interpretation/importance to this result?