MIDTERM 2

Physics 219A, Spring 2014

Do four of five problems!

[**1.**] Compute, by explicit enumeration of all the states, the partition function $Z(T)$ and average energy $\langle E \rangle(T)$ of the $N = 6$ site one dimensional Ising model with periodic boundary conditions. Verify your expression for Z by checking against the exact solution $Z = (2\cosh\beta J)^N + (2\sinh\beta J)^N$. If you find $N = 4$ too hard to do, try $N = 4$.

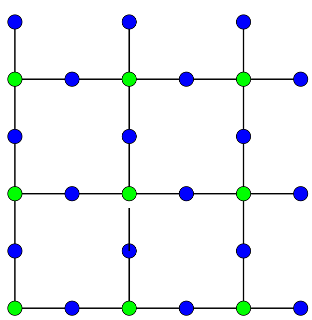
[**2.**] Make qualitative plots of the susceptibility $\chi(T)$, and the magnetization $m(T)$ at field $B = 0$ for a system with a second order magnetic phase transition. How are the critical exponents β and γ defined? Make qualitative plots of the magnetization $m(B)$ for $T > T_c$, for $T = T_c$ and for $T < T_c$. How is the critical exponent δ defined? What are the Mean Field values of β, γ, and δ? Discuss any relationships between the plots that occur to you. For example, does the behavior of $\chi(T)$ near T_c seem plausible in terms of what happens to $m(T)$?

[**3.**] Write down the high and low T expansions for the partition function of the Ising model on a hexagonal lattice. For high T go to order ten in $\tanh\beta J$. For low T go to order four in $\exp(-2\beta J)$.

[**4.**] Landau has a nice way of picturing any phase transition. What is it?

[**5.**] Suppose someone challenges you to work out Mean Field Theory (MFT) for the Ising model on the "Lieb Lattice". See below. Unlike the square lattice we did in class, where all the sites are identical, there are two different types of sites here (one type has two neighbors and the other has four neighbors). How might you generalize MFT to such a situation? How would you formulate self-consistency? How would you go about determining T_c?

Hint: Allow $\langle S_i \rangle$ to be different depending on which type of site is being considered.



The Lieb lattice.