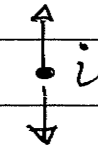
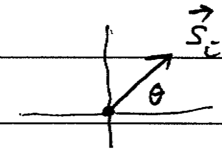
Other Models of Magnetism

Ising $\quad E = -J \sum_{\langle ij \rangle} S_i S_j \qquad S_i = \pm 1$

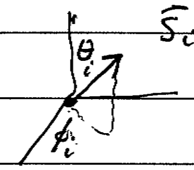


XY $\quad E = -J \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y \right)$

$\cos\theta_i \quad \cos\theta_j \quad \sin\theta_i \quad \sin\theta_j$



$$= -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$



Heisenberg $\quad E = -J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$

$$= -J \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z \right)$$

Notice Each of these models is defined on any spatial lattice you like. The dimensionality of the spin variable (# of components) has <u>nothing</u> to do with the dimensionality of the lattice.

NB this is <u>classical</u> Heisenberg model.
can also make $\hat{S}_i^x, \ldots$ into <u>operators</u>.

M-2

dim of spatial lattice — Why?

As d increases, ordering tendency is more pronounced

As n increases, ordering tendency is less pronounced

# of spin components — Why?

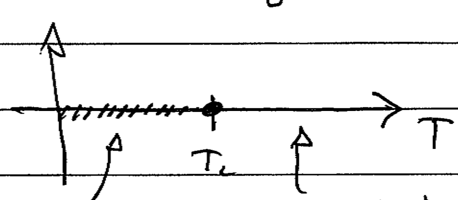
TABLE

⟶ less order

d	$n=1$ (Ising)	$n=2$ (XY)	$n=3$ (Heisenberg)
1	no transition $T_c = 0$	no transition $T_c = 0$	no transition $T_c = 0$
2	transition! $T_c = 2.269J$		no transition $T_c = 0$
3	transition! $T_c = 4.51J$	transition! $T_c = 2.20J$	transition! $T_c = 1.43J$

↓ more order

funny "Kosterlitz Thouless" transition



C(r) decays exponentially

C(r) decays as power law $\forall T < T_c$ No true long range order

"line of critical points"

M-3

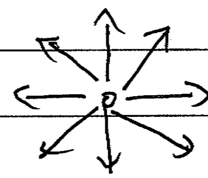
p-State

"Potts Model" $S_i = 1, 2, \ldots p$ (Ising is basically the case $p = 2$)

$$E = -J \sum_{\langle ij \rangle} \delta_{S_i, S_j}$$

~~What can you guess about T_c relative to Ising?~~

Clock model



2D spin with discrete directions instead of continuous θ_i