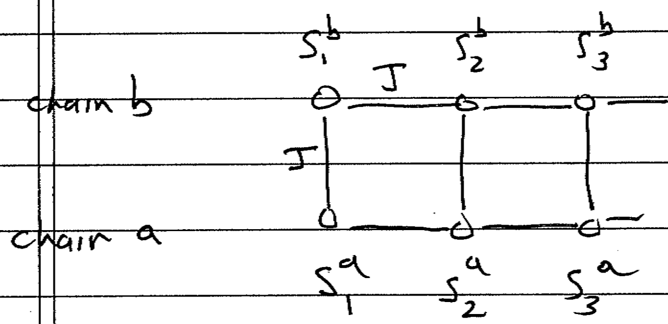
D1

Some Notes from Discussion Section

1. How would you do 2D Ising by transfer matrix?

Consider two chains:



$$E = -J \underbrace{\sum_{\langle ij \rangle} S_i S_j}$$

includes intra and inter chain neighbours

$$Z = \sum_{S_\ell^a} \sum_{S_\ell^b} e^{+\beta J \sum_{\langle ij \rangle} S_i S_j}$$

$$= \sum_{S_\ell^a} \sum_{S_\ell^b} \underbrace{e^{\beta J S_1^a S_2^a} e^{\beta J S_1^b S_2^b}}_{\text{intrachain}} \underbrace{e^{\beta J S_1^a S_1^b} e^{\beta J S_2^a S_2^b}}_{\text{interchain}} \cdots$$

$$M(\underbrace{S_1^a S_1^b}_{\text{row}} | \underbrace{S_2^a S_2^b}_{\text{column}}) \equiv e^{\beta J S_1^a S_2^a} e^{\beta J S_1^b S_2^b} e^{\beta J S_1^a S_1^b/2} e^{\beta J S_2^a S_2^b/2}$$

4 choices for each!

why 1/2?

$Z = \mathrm{Tr}\, M^N$ by usual argument

Now M is 4×4 matrix

D2

$\xrightarrow{\text{col } S_2^a S_2^b}$

$$
\begin{array}{c c}
\downarrow \text{row} & \\
S_1^a S_1^b & \begin{array}{cccc} ++ & +- & -+ & -- \end{array} \\
\begin{array}{c} ++ \\ +- \\ -+ \\ -- \end{array} &
\begin{pmatrix}
e^{3\beta J} & 1 & 1 & e^{\beta J} \\
1 & e^{\beta J} & e^{-3\beta J} & 1 \\
1 & e^{-3\beta J} & e^{\beta J} & 1 \\
e^{\beta J} & 1 & 1 & e^{3\beta J}
\end{pmatrix}
\end{array}
$$

Compute 4 eigenvalues

$$Z = \lambda_1^N + \lambda_2^N + \lambda_3^N + \lambda_4^N$$

Now do 3, 4, 5, .. chains.

How big with computer? Recall Phys 215B

CPU time $\sim \underbrace{N^3 / 10^9}_{\text{GHz chip}} \sim \underbrace{10^4 \text{ sec}}_{2 \text{ hours}}$

$N^3 \sim 10^{13} \quad N \sim 20000$

Memory $\sim$ 10 Gbytes $\sim 10^{10} \sim N^2 \, \underbrace{10}_{\text{64 bit \# = 8 bytes}}$

$N^2 \sim 10^9 \quad N \sim 30000$

P3

$$2^{N_{chain}} = 20000$$

$$2^{14} = 16384$$ so 14 chains

I have actually never done this problem.

I would be interested in seeing it solved

Specifically, compare $\langle E \rangle_{onsager}$ to $\langle E \rangle_{14\ chains}$

Actually, only need largest λ in $N \to \infty$ limit.

"Lanczos" method can do up to $\sim 10^6$ dim matrix

$$2^{N_{chain}} = 10^6$$

$$N_{chain} = 20$$

D4

② Ising Model as a description of melting

We allowed any spin configuration

Suppose we allow any spin configuration restricted to

$M = \sum s_i = 0 \Leftarrow$ ie equal # of $\uparrow, \downarrow$.

low T looks like this : 2 ferromagnetic clumps is best we can do. to minimize E.

```
+ + + + | - - - -
+ + + + | - - - -
+ + + + | - - - -
+ + + + | - - - -
```

"domain wall"

high T is usual "jumble"

```
+ - - + -
- - + - +
+ + - + -
- - + - -
```

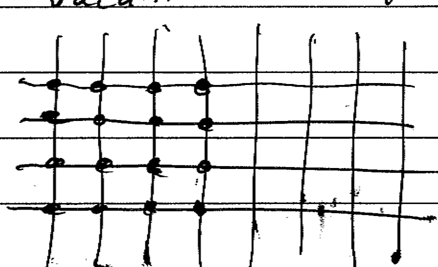
Now, consider defining $n_i = \frac{1}{2}(s_i + 1)$

n_i takes values $\emptyset, 1$ and can be interpreted as

particle present on site $n_i = 1$ $(s_i = +1)$

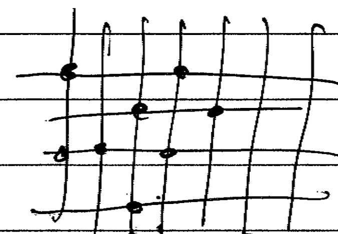
~~p~~ site is vacant $n_i = 0$ $(s_i = -1)$

Mapping above "spin pictures" ~~+~~ particle picture



low T solid

Melting $\longleftrightarrow$



high T liquid