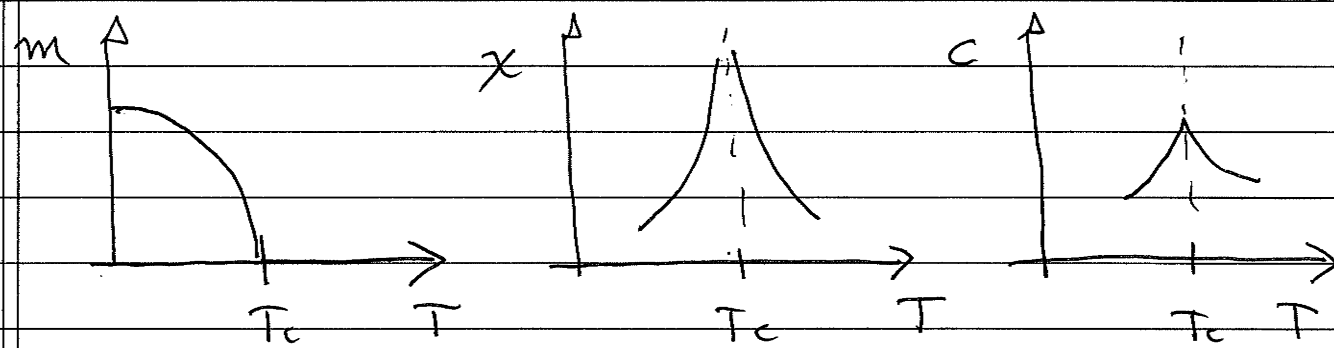
CF1

Correlation Functions and Phase transitions

We know



$(T_c - T)^\beta$ $\qquad$ $1/(T-T_c)^\gamma$ $\qquad$ $1/(T-T_c)^\alpha$

For $d=1$ Ising $\langle S_1 S_{1+\ell} \rangle \sim e^{-\ell/\xi}$

$$\xi = -\ln(\tanh \beta J)$$

no phase transition.

In general, when there is a phase transition

$$c(\vec{r}) \equiv \langle S_{\vec{r}_0} S_{\vec{r}_0+\vec{r}} \rangle \sim e^{-|\vec{r}|/\xi} \qquad T > T_c$$

$$\sim 1/|\vec{r}|^p \qquad T = T_c$$

$$\sim \text{const} \neq 0 \qquad T < T_c$$

CF2

Behavior of correlation function related to

that of susceptibility χ (per site)

$$\chi = \frac{d\langle M\rangle}{dB} = \frac{1}{N}\beta\left[\langle M^2\rangle - \langle M\rangle^2\right]$$

sites $\quad M = \sum_{\vec{r}} S_{\vec{r}}$

Above T_c $\quad \langle M\rangle = 0$ and

$$\langle M^2\rangle = \langle \sum_{\vec{r}} \sum_{\vec{r}\,'} S_{\vec{r}} S_{\vec{r}\,'} \rangle$$

$$= N \sum_{r} \langle S_{\vec{r}_0} S_{\vec{r}_0+\vec{r}} \rangle$$ by translation invariance

$$= N \sum_{r} c(\vec{r})$$

If $c(\vec{r})$ decays exponentially $\quad \langle M^2\rangle \sim N$

and $\chi \sim \frac{1}{N}\beta N \int r^2 dr\, 4\pi e^{-r/\xi} \sim$ indep of N (nondivergent)

But if $c(\vec{r})$ decays as power law, eg at T_c

$$\chi = \frac{1}{N}\beta N \underbrace{\int_0^\infty r^2 dr\, 4\pi \, 1/r^p}_{\text{diverges unless } p>3}$$

Conclude power laws behavior of $C(\vec{r})$ linked to divergence of χ at T_c