$\boxed{\text{Dirac Eqn}}$

Schroedinger Eqn

$$+i\hbar \frac{\partial \psi(rt)}{\partial t} = \frac{1}{2m}\left(-\frac{\hbar}{i}\nabla - \frac{e}{c}\vec{A}\right)^2 \psi(rt) - e\phi(rt)\psi(rt)$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}$$

Treats space (r) and time (t) differently.

Dirac Eqn (BD55)

$$i\hbar \frac{\partial \psi(rt)}{\partial t} = \left\{ c\,\vec{\alpha}\cdot\left(\frac{\hbar}{i}\nabla + \frac{e}{c}\vec{A}\right) - e\phi + \beta mc^2 \right\}\psi(rt)$$

More specifically, Can show it is Lorentz invariant

Space & time both appear as first derivatives, ie symmetrically. But added complication ψ is a column vector of $\boxed{\text{four}}$ functions, and α, β are 4×4 matrices

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \qquad \vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

2×2 Pauli matrices

2×2 identity matrix

D2

There's a whole long rationale for this form (why matrices and multicomponent ψ are necessary for Lorentz-invariant theory) but I cannot go into that.

Likewise there are nice problems to be done: Solve Dirac eqn for free fermions $\vec{A} = \phi = 0$, for hydrogen atom, etc.

In 215AB (nonrelativistic QM) to get energy $E = \langle \psi | H | \psi \rangle$ we could work in a particular basis like position representation

$$\int d^3r\ \psi^*(rt) \left\{ \frac{1}{2m} \left(-\frac{\hbar}{i} \vec{\nabla} - \frac{e}{c} \vec{A} \right)^2 - e\phi \right\} \psi(rt)$$

~~(BD55)~~

Analog here is

$$\int d^3r\ \psi^\dagger(rt) \left\{ c\vec{\alpha} \cdot \left(\frac{\hbar}{i} \nabla + \frac{e}{c} \vec{A} \right) - e\phi + \beta mc^2 \right\} \psi(rt)$$

$$\underbrace{\int d^3r\ \psi^\dagger(rt) \left\{ c\alpha \cdot \frac{\hbar}{i} \nabla + \beta mc^2 \right\} \psi(rt)}_{H_0} + \underbrace{\int d^3r\ \psi^\dagger(rt) \left\{ \quad \right\} \psi(rt)}_{H_{int}}$$

D3

Recall from 215B $\hat{H} = \hat{H}_0 + \hat{H}_{int}$

$$e^{-i/\hbar \hat{H} t} = e^{-i/\hbar \hat{H}_0 t}\, T\exp\left\{ -\frac{i}{\hbar} \int_{-\infty}^{t} H_{int}(t')\,dt' \right\}$$

We are now in a position to give a schematic description of connection between stat mech and QFT

STAT MECH $\langle S_1 S_{1+e} \rangle = Z^{-1} \sum e^{-\beta H} S_1 S_2$

spin spin correlation function

$\downarrow$

High T expansion

$\downarrow$

Diagrams $\begin{cases} \text{connected} \\ \text{disconnected} \end{cases}$

~~Various rules for~~ what diagrams are nonzero

cancelation of disconnected diagrams ($O(N^2)$ terms) etc etc.

D4.

(BD 135)

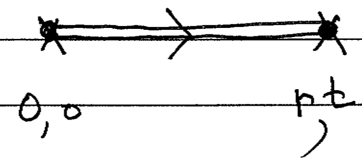
QFT $\langle \psi(rt)\psi^\dagger(0,0)\rangle = Z^{-1}\langle T\exp\left(\frac{-i}{\hbar}\int H_{int}(t')dt'\right)\psi(rt)\psi^\dagger(00)\rangle$

Greens function, propagator

full H

power series expand (analog of high T expansion) but more obvious

H_0 only



$$\mathbb{I} - \frac{i}{\hbar}\int H_{int}(t')dt' + \ldots$$

$$\mathbb{I} - \frac{i}{\hbar}\int d^3r'\int dt'\ \psi^\dagger(r't')\left(e\vec{\alpha}\cdot\vec{A} - e\phi\right)\psi(r't') + \ldots$$

get objects like

$$\langle\ \psi(rt)\ \psi^\dagger(00)\ \psi^\dagger(r't')\,A(r't')\,\psi(r't')\ \rangle$$

Rules for getting non zero answer from Feynman $\longleftrightarrow$ analog of closed loop requirement in our high T expansion

must pair ψ up with $\psi^\dagger$

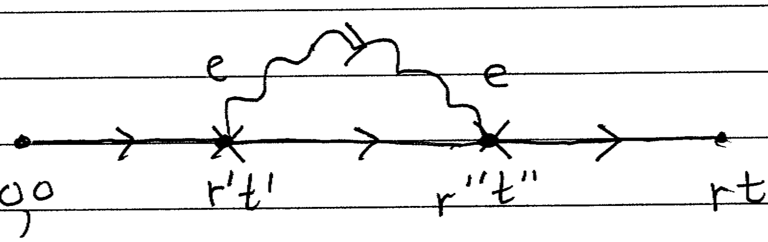
and A with another A

not enough here

D5

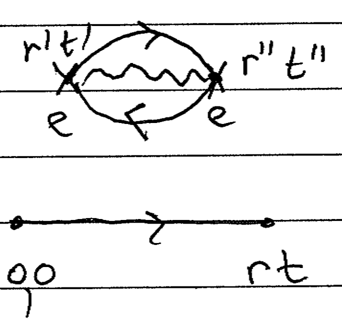
Need next term in $T\exp -\frac{i}{\hbar}\int H_{int}(t')\,dt'$

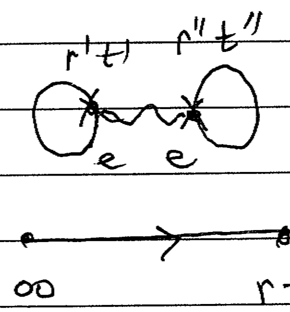
$$e^2\langle \psi(rt)\psi^\dagger(00)\,\psi^\dagger(r't')A(r't')\psi(r't')\,\psi^\dagger(r''t'')A(r''t'')\psi(r''t'')\rangle$$



other pairings are possible!

$$e^2\langle \psi(rt)\,\psi^\dagger(00)\,\psi^\dagger(r't')A(r't')\psi(r't')\psi^\dagger(r''t'')A(r''t'')\psi(r''t'')\rangle$$





Can ignore disconnected diagrams because such "vacuum fluctuations" exist even in absence of $\psi(rt)\psi^\dagger(00)$ process.

This is what mysterious "Z" is for in QFT

D6

Analogy table

Stat Mech	QFT
spin-spin correlation function $\langle S_1 S_{1+\ell} \rangle$	propagator $\langle \psi(rt)\psi^\dagger(00)\rangle$
Boltzmann factor $e^{-\beta E}$	time ordered exponential $Te^{-\frac{i}{\hbar}\int H_{int}(t')dt'}$
high T expansion	Taylor expansion
Rules for nonzero diagrams	(Wick's Theorem) rules for nonzero diagrams
~~cancellation of classes of nonzero diagrams~~ (disconnected ones which would give non extensive free energy)	cancellation of ~~disconnected~~ diagrams