PROBLEM SET 6– due Thurs. 6-12

Physics 215C- Quantum Mechanics, SPRING 2008

In all problems of this assignment, set $\hbar = 1$.

1. A particle with spin-1 sits in a magnetic field $\vec{B} = (-\sqrt{2}t, 0, -h)$ so that,

$$\hat{H} = -h\,\hat{L}_z - \sqrt{2}\,t\,\hat{L}_x \ ,$$

where $\hat{L}_x$ and $\hat{L}_z$ are spin-1 angular momentum matrices. Compute the partition function if the particle is in thermal equilibrium at inverse temperature β. Compute $\langle L_z \rangle$. What are the high and low temperature limits. *Thought question*: Do you notice a relationship between the eigenvalues and the changes in sign in the eigenvectors? Do you know a general rule which would have led you to expect this relationship?

2. By following the derivation of the path integral for the real time propagator $\langle x|e^{-i\hat{H}t/\hbar}|x'\rangle$ presented in class, derive the form for $S(x_1, x_2, \cdots x_L)$ in the expression the partition function of a quantum particle moving in one dimension,

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$$
$$Z = \mathrm{Tr}\, e^{-\beta\hat{H}} = \int dx_1 \int dx_2 \cdots \int dx_L \ e^{-S(x_1, x_2, \cdots x_L)} \ .$$

3. Show that if $V(\hat{x}) = \frac{1}{2}m\omega^2\hat{x}^2$ then $S = \mathbf{x}\,\mathcal{M}\,\mathbf{x}^T$ where $\mathbf{x} = (x_1, x_2, \cdots x_L)$ and $\mathcal{M}$ is an matrix of dimension L. What are the elements of $\mathcal{M}$?

4. Do all the integrals in problem two for the V of problem three. Use the expressions for multi-dimensional Gaussian integrals discussed in class.

5. Continue problem three and compute $\langle\, \hat{x}^2 \,\rangle$. Again, use the expressions for multi-dimensional Gaussian integrals discussed in class. You do not need to evaluate the sum (but see next problem).

6. <u>Optional, Extra Credit</u>: (Set $k_{\mathrm{B}} = 1$ in this problem.) Evaluate the sum in problem four for $m = 1$, $\omega = 2$, and $T = 0.5$. (That is, $\beta = 2$). Use $L = 4$, $L = 8$ and $L = 16$ and compare with the exact $\langle \hat{x}^2 \rangle$.