MIDTERM EXAM

Physics 215C- Quantum Mechanics, SPRING 2008

Do three of the four problems. (Fourth problem is on reverse side.)

1. A system with unperturbed eigenstates $\phi_n(\vec{x})$ and energies E_n is subject to a perturbation,

$$H'(t) = \frac{A}{\sqrt{\pi}\tau}\, e^{-t^2/\tau^2},$$

where A is a time independent operator.

a. If at $t = -\infty$ the system is in its ground state ϕ_0, show that, to first order, the probability amplitude that at $t = +\infty$ the system will be in its m–th excited state ($m \neq 0$) is,

$$c_m(+\infty) = -i\frac{\langle m|A|0\rangle}{\hbar} e^{-\frac{\tau^2}{4\hbar^2}(E_0-E_m)^2}.$$

b. Next consider the limit of an impulsive perturbation ($\tau \to 0$) and compute the probability that the system makes any transition out of its ground state.

2. Compute the scattering cross section from a delta function potential $V(\vec{r}) = V_0\,\delta(\vec{r})$ in the Born approximation.

3. A quantum mechanical system with a two dimensional Hilbert space has a Hamiltonian,

$$H_0 = E\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

At time $t = 0$ the system is known to have a wavefunction $\psi(t=0) = \binom{1}{0}$, in the same basis in which H_0 is written.

a. What are the energy eigenvectors and eigenvalues?

b. What is $\psi(t)$ for times $t \geq 0$?

c. What are the probabilities the system is in the ground state and first excited state at $t = \infty$?

d. A perturbation,

$$V(t) = \lambda E e^{-2tE/\hbar}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}.$$

is applied, starting at $t = 0$. (That is, $V = 0$ for $t < 0$.) Here λ is a small number. Determine the lowest order corrections to probabilities of part (c).

4. Short Answers Only!

   a. A charged particle moves in a vector potential $\vec{A} = Bx\,\hat{y}$. Write down, but do not solve, the Schroedinger equation the wave function satisfies.

   b. What is "Fermi's Golden Rule"?

   c. If $\Psi(x)$ is a solution of the Schroedinger equation, then so is $e^{i\phi}\,\Psi(x)$, where ϕ is a constant phase. What can we do to Schroedinger's equation so that $e^{i\phi(x)}\,\Psi(x)$ will be a solution when $\Psi(x)$ is?

   d. Suppose $\hat{H} = \hat{H}_0 + \hat{V}$. In the Schroedinger picture $|\,\Psi(t)\,\rangle = e^{-i\hat{H}t/\hbar}\,|\,\Psi(0)\,\rangle$. In the Heisenberg picture $|\,\Psi(t)\,\rangle = |\,\Psi(0)\,\rangle$. What is $|\,\Psi(t)\,\rangle$ in the interaction picture? (It is fine if you just give the initial definition. You do not need to derive the time ordered exponential and all that.)