
Problem Set 3

(Due Friday, Feb. 13.)

Problem 1: Solve the problem of a quantum mechanical particle moving in three dimensions with the central potential \( V(x,y,z) = \frac{1}{2} m \omega^2 (x^2 + y^2 + z^2) = \frac{1}{2} m \omega^2 r^2 \), by working in spherical coordinates. I.e. do not write the solution as the separable product of \( x, y, z \) wave–functions, but instead divide the wavefunction into radial and angular parts, and solve the radial equation. Work out the degeneracies of the energy levels and show they agree with a solution based on using Cartesian coordinates.

Problem 2: In class we solved the Hydrogen atom problem \( H \psi = E \psi \) where \( H = (p_x^2 + p_y^2 + p_z^2)/2m - e^2/r \). The answer was
\[
\psi(r,\theta,\phi) = R_{n,l}(r)Y_{m}^{l}(\theta, \phi).
\]
Here \( Y_{m}^{l}(\theta, \phi) \) were the spherical harmonics and \( R_{n,l}(r) \) was the radial wave function, the solution to some one dimensional differential equation. Also \( E = -me^4/(2\hbar^2 n^2) \), with \( n \) a positive integer. Compute the eigenfunctions and eigenvalues of the Hydrogen atom with an additional term proportional to \( L_z \), that is \( H = (p_x^2 + p_y^2 + p_z^2)/2m - e^2/r + AL_z \), where \( A \) is some constant. Hint: This problem can be done without much work.

Problem 3: A one–dimensional quantum harmonic oscillator is perturbed by an extra potential \( \lambda x^4 \). Calculate the eigenvalues to first order in \( \lambda \).

Problem 4: The Hamiltonian of a rigid rotator in a constant magnetic field in the \( xz \) plane is
\[
H = \frac{1}{2I} L^2 + a_z L_z + a_x L_x.
\]
Suppose \( a_x << a_z \). Find the eigenvalues of \( H \) to second order in \( a_x \). Compare with the exact results.