Physics 215B, Winter 1998

Problem Set 3

(Due Friday, Feb. 13.)

Problem 1: Solve the problem of a quantum mechanical particle moving in three dimensions with the central potential $V(x,y,z) = \frac{1}{2}m\omega^2(x^2+y^2+z^2) = \frac{1}{2}m\omega^2 r^2$, by working in spherical coordinates. *I.e.* do not write the solution as the separable product of x, y, z wave–functions, but instead divide the wavefunction into radial and angular parts, and solve the radial equation. Work out the degeneracies of the energy levels and show they agree with a solution based on using Cartesian corrdinates.

Problem 2: In class we solved the Hydrogen atom problem $H\psi = E\psi$ where $H = (p_x^2 + p_y^2 + p_z^2)/2m - e^2/r$. The answer was

$$\psi(r,\theta,\phi) = R_{n,l}(r)Y_l^m(\theta,\phi).$$

Here $Y_l^m(\theta,\phi)$ were the spherical harmonics and $R_{n,l}(r)$ was the radial wave function, the solution to some one dimensional differential equation. Also $E = -me^4/(2\hbar^2 n^2$, with n a positive integer. Compute the eigenfunctions and eigenvalues of the Hydrogen atom with an additional term proportional to L_z, that is $H = (p_x^2 + p_y^2 + p_z^2)/2m - e^2/r + AL_z$, where A is some constant. Hint: This problem can be done without much work.

Problem 3: A one–dimensional quantum harmonic oscillator is perturbed by an extra potential λx^4. Calculate the eigenvalues to first order in λ.

Problem 4: The Hamiltonian of a rigid rotator in a constant magnetic field in the xz plane is

$$H = \frac{1}{2I}L^2 + a_z L_z + a_x L_x.$$

Suppose $a_x << a_z$. Find the eigenvalues of H to second order in a_x. Compare with the exact results.