FINAL EXAM

Physics 215B- Quantum Mechanics, WINTER 1998

Problem 1: This multiple choice question is in honor of Jason Trento. Suppose a particle approaches a scattering center with a trajectory of impact parameter "b", and is then scattered through an angle θ. (See figure.) What is the expected sign of $d\theta/db$?
(a) Positive.
(b) Zero.
(c) Negative.

Problem 2: You are given two objects of angular momentum j_1 and j_2. Denote by $|\, j_1\, j_2\, m_1\, m_2\,\rangle$ the vector which is an eigenvector of the operators $(\hat{J}_1)^2$, $(\hat{J}_2)^2$, $\hat{J}_1^z$, and $\hat{J}_2^z$. That is,

$$\begin{aligned}
(\hat{J}_1)^2|\, j_1\, j_2\, m_1\, m_2\,\rangle &= j_1(j_1+1)\hbar^2|\, j_1\, j_2\, m_1\, m_2\,\rangle \\
(\hat{J}_2)^2|\, j_1\, j_2\, m_1\, m_2\,\rangle &= j_2(j_2+1)\hbar^2|\, j_1\, j_2\, m_1\, m_2\,\rangle \\
\hat{J}_1^z|\, j_1\, j_2\, m_1\, m_2\,\rangle &= m_1\hbar|\, j_1\, j_2\, m_1\, m_2\,\rangle \\
\hat{J}_2^z|\, j_1\, j_2\, m_1\, m_2\,\rangle &= m_2\hbar|\, j_1\, j_2\, m_1\, m_2\,\rangle.
\end{aligned}$$

(a) How many such vectors are there?
(b) What are the possible eigenvalues of the square of the total angular momentum $(\hat{J}_1+\hat{J}_2)^2$?
(c) Denote by $|\, j\, j_1\, j_2\, m\,\rangle$ the vector which is an eigenvector of the operators $(\hat{J}_1+\hat{J}_2)^2$, $(\hat{J}_1)^2$, $(\hat{J}_2)^2$, $\hat{J}_1^z+\hat{J}_2^z$. Show that the number of such states is the same as in (a). (Hint: $\sum_{j=1}^{n} j = n(n+1)/2$.)

Problem 3: A particle moves in two dimensions in the potential, $V(x,y)=0$ if $|x|<a$ and $|y|<b$, $V(x,y)=\infty$ otherwise. **(a)**What are the energy eigenfunctions and eigenvalues?
(b) The potential is modified to include an additional term λxy. Write an expression for the first order energy shift of the ground state *but do not evaluate the integrals*!!
(c) State clearly what you need to do to calculate the first order energy shift of the first excited state in the case $a=b$, but, again, do not evaluate any integrals.

Problem 4: Calculate $f_k(\theta,\phi)$ in the Born approximation for scattering off the Yukawa potential $V(r)=V_0\, e^{-\alpha r}/r$.

Problem 5: The component of spin in the $\hat{u}=(1/\sqrt{2},0,1/\sqrt{2})$ direction of a spin–1/2 particle is measured, and found to be $+\hbar/2$. What is the wavefunction $|\psi\rangle$ just after the measurement?

Problem 6: Find the energy eigenvalues and eigenfunctions of H_0 where

$$H_0 = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} \tag{1}$$

$$V = \alpha \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}. \tag{2}$$

Compute the shift of the first excited state to second order due to the perturbation V. What is the exact first excited state energy? Do your answers agree?