Physics 215B- Quantum Mechanics, Winter 2014

Problem Set 5, due Thursday March 6

[**1.**] *Qualifier Problem!* A system with unperturbed eigenstates $\phi_n(x)$ and energies E_n is subject to a perturbation

$$\hat{V} = \frac{\hat{A}}{\sqrt{\pi}\tau} e^{-t^2/\tau^2}$$

where $\hat{A}$ is a time-independent operator.

a) If at $t = -\infty$ the system is in its ground state ϕ_0, show that, to first order, the *probability amplitude* that at $t = +\infty$ the system will be in its m-th excited state ($m \neq 0$) is:

$$c_m(+\infty) = -i \, \frac{\langle \, m \, | \, \hat{A} \, | \, 0 \, \rangle}{\hbar} \exp\left(-\frac{\tau^2}{4\hbar^2}(E_0 - E_m)^2 \right)$$

b) Next consider the limit of an impulsive perturbation $\tau = 0$ and compute the *probability* P that the system makes any transition out of the ground state.

[**2.**] *Qualifier Problem!* A particle of charge e and mass m is bound in a three dimensional harmonic oscillator potential $V = \frac{1}{2}m\omega^2 r^2$. We wish to calculate the lifetime of the first excited state.

a) Show that the condition for the electric dipole approximation to be valid is $\hbar\omega << mc^2$.

b) Calculate the lifetime of the first excited state in this limit.

c) The first excited state is degenerate. What effect (if any) does this have on the calculation in part b above?

[**3.**] *Qualifier Problem!* A one microampere beam of 100 keV electrons is incident on a 1 cm thick target of hydrogen of density 10^{-3} g/cm^3. Approximate the interaction potential between an electron and a hydrogen atom by the screened Coulomb interaction

$$V(r) = -\frac{e^2}{r} e^{-r/a_0}$$

where a_0 is the Bohr radius. Using the first Born approximation, calculate the number of electrons per second scattered elastically into a detector at an angle of 10 mrad to the incident beam and which subtends a solid angle of 10^{-5} steradians.

[**4.**] *Qualifier Problem!* Consider the scattering of a particle of mass m off a spherical well of radius R and depth V_0.

a) Find the total cross section for the scattering of very low energy particles off the well.

b) Find the total cross section for very high energies.

You may leave your answers in terms of precisely defined integrals of known functions.