Physics 215B- Quantum Mechanics, Winter 2014

Problem Set 4, due Tuesday February 18

[**1.**] Consider the quantum harmonic oscillator, and perturbation

$$\hat{H}_0 = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\,\omega_0^2\,\hat{x}^2 \qquad\qquad \hat{V} = \delta\,\hat{x}^2$$

Compute the first and second order shifts in the energy levels of $\hat{H}_0$ due to $\hat{V}$. If you have time, solve the problem exactly and expand your result in a Taylor series in δ to check the perturbation calculation. You may find the identity $\hat{x} = \sqrt{\hbar/(2m\omega_0)}\,(\,a\ + a^\dagger\,)$ useful.

[**2.**] *Qualifier Problem!* Solve for and the sketch the perturbed energy levels of the $n = 2$ level of hydrogen in the presence of both a uniform electric and a uniform magnetic field. Neglect the spin of the electron and consider only orbital degrees of freedom. Do this for two cases:
(a) The $\vec{E}$ and $\vec{B}$ fields are parallel in orientation.
(b) The $\vec{E}$ and $\vec{B}$ fields are at right angles to each other.
You may find the following relations useful: $\langle 2s|x|2p, m = \pm 1\rangle = (3/\sqrt{2})a_0$, and $\langle 2s|z|2p, m = 0\rangle = 3a_0$, *Note:* We may need to review how to put a magnetic field $\vec{B}$ into a quantum mechanical Hamiltonian. What you do is replace $\hat{p}$ by $\hat{p} - (e/c)\vec{A}$ where A is the vector potential which produces the desired $\vec{B} = \nabla \times \vec{A}$.

[**3.**] *Qualifier Problem!* Consider an atom in a crystal with two eigenstates ψ_1 and ψ_2 with energies $E_{1,0}, E_{2,0}$. As the crystal is compressed, the crystalline field varies and the eigenstates are perturbed. The matrix of the perturbation in the basis of ψ_1 and ψ_2 is

$$\hat{H} = \begin{pmatrix} 0 & 3\epsilon \\ 3\epsilon & 4\epsilon \end{pmatrix}$$

(a) What are the first and second order corrections to the energies of the states if $E_{1,0} \neq E_{2,0}$?
(b) What are the lowest order corrections to the energies if $E_{1,0} = E_{2,0}$?
(c) Sketch the energies of the two states as a function of the compression parameter ϵ.

[**4.**] Consider the three dimensional infinite cubical well, $V_0(x, y, z) = 0$ if $0 < x, y, z < a$ and $V_0(x, y, z) = \infty$ otherwise. Add a perturbation $V(x, y, z) = W$, if $0 < x, y < a/2$ and $V(x, y, z) = 0$ otherwise. Compute the shifted eigenenergies and eigenstates of the ground state and first excited states.

[**5.**] Compute, to first order, the correction to the ground state energy of a Hydrogen atom due to the finite spatial extent of the nucleus. For simplicity, assume the nucleus is spherical, of radius R, and that its charge e is uniformly distributed throughout its volume.

[**6.**] Compute, to first order, the changes to the energy levels of a Hydrogen-like atom (ie one with nuclear charge Ze) produced by a unit increase $Z \to Z + 1$ (due, for example, to β decay). Using the exact energies, discuss the validity of the approximation.