PTFT-1

Perturbation Theory in Field Theory (Feynman Diagrams)

Fundamental identity is "Feynman Disentangling Thm"

$$\hat{H} = \hat{H}_0 + \hat{V}$$

$$e^{-i\hat{H}t} = e^{-i\hat{H}_0 t}\, \mathrm{T}\exp\left(-i\int_0^t \hat{V}(t')\,dt'\right)$$

"time ordered exponential" ?!?

$$\hat{V}(t') = e^{i\hat{H}_0 t}\, \hat{V}\, e^{-i\hat{H}_0 t}$$

$$e^{-i\hat{H}t} = e^{-i\hat{H}_0 t}\, \hat{\mathcal{U}}(t)$$

time evolution due to $\hat{H}_0$

"small" ie close to I operator

$$\hat{\mathcal{U}}(t) = e^{i\hat{H}_0 t}\, e^{-i\hat{H}t}$$

$$\frac{d\hat{\mathcal{U}}}{dt} = e^{i\hat{H}_0 t}\, \hat{H}_0\, e^{-i\hat{H}t} - e^{i\hat{H}_0 t}\, \hat{H}\, e^{-i\hat{H}t}$$

$$= e^{i\hat{H}_0 t}\, \hat{V}\, e^{-i\hat{H}t}$$

$$= -i e^{i\hat{H}_0 t}\, \hat{V}\, e^{-i\hat{H}_0 t}\, e^{i\hat{H}_0 t}\, e^{-i\hat{H}t}$$

$$= -i\hat{V}(t)\, \hat{\mathcal{U}}(t)$$

Convert this differential eqn into integral eqn

$$\hat{U}(t) = \mathbb{I} - i\int_0^t \hat{V}(t_1)\hat{U}(t_1)\,dt_1$$

$\hat{U}(t=0) = \mathbb{I}$ (see $e^{-i\hat{H}t} = e^{-i\hat{H}_0 t}\,U(t)$

can also do algebra with

$\hat{U}(t_0) = \mathbb{I}$)

Solve iteratively

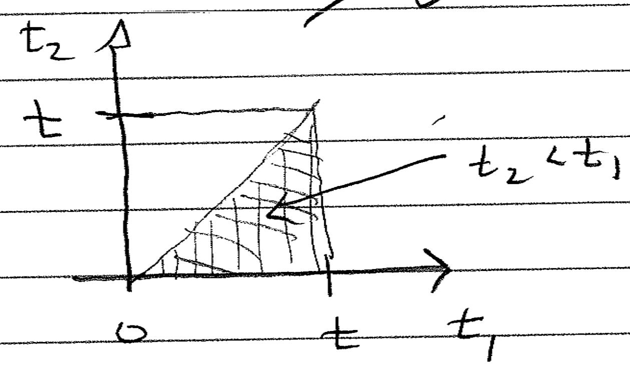
$$\hat{U}_0(t) = \mathbb{I}$$

plug in $\hat{U}(t_1) = \mathbb{I}$

$$\hat{U}_1(t) = \mathbb{I} - i\int_0^t \hat{V}(t_1)\,dt_1$$

$$U_2(t) = \mathbb{I} - i\int_0^t dt_1\,\hat{V}(t_1)\left\{\mathbb{I} - i\int_0^{t_1}\hat{V}(t_2)\,dt_2\right\}$$

$$= \mathbb{I} - i\int_0^t dt_1\,\hat{V}(t_1) + (-i)^2\int_0^t dt_1\int_0^{t_1} dt_2\,\hat{V}(t_1)\hat{V}(t_2)$$



By symmetry last term is

$$(-i)^2 \; \frac{1}{2} \int_0^t dt_1 \int_0^t dt_2 \quad \hat{V}(t_1)\hat{V}(t_2)$$

$\boxed{\text{BUT}}$ in original integral $\hat{V}(t_1)\hat{V}(t_2)$

$$\underset{\text{later}}{\uparrow} \qquad \underset{\text{earlier}}{\uparrow}$$

So need really

$$(-i)^2 \quad \frac{1}{2} \int_0^t dt_1 \int_0^t dt_2 \underset{\substack{\uparrow \\ \text{time ordering operator}}}{T} \hat{V}(t_1)\hat{V}(t_2)$$

→ latest time to left

So $e^{-i\hat{H}t} = e^{-i\hat{H}_0 t} \, T\exp -i\int_0^t \hat{V}(t_1)dt_1$

* This calculation is a preview of Physics 230, 240C

also hints at importance/usefulness of converting

differential eqn to integral eqn

$$\frac{d\hat{u}}{dt} = -i\hat{V}(t)\hat{u}(t) \qquad \hat{u}(t) = I - i\int_0^t \hat{V}(t_1)\,\hat{u}(t_1)\,dt_1$$