TD1

"periodic bc"

$$H = \begin{bmatrix} A & B & 0 & 0 & 0 & & B \\ B & A & B & 0 & 0 & & \\ 0 & B & A & B & 0 & & \\ 0 & 0 & B & A & B & & \\ & & & & & \ddots & \\ B & & & & & & \end{bmatrix} \leftarrow \text{dim } N$$

Q: What are eigenvalues and eigenvectors?

$$H|\psi\rangle = E|\psi\rangle \qquad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix}$$

$$B\psi_{\ell-1} + A\psi_\ell + B\psi_{\ell+1} = E\psi_\ell$$

guess $\psi_\ell = c e^{ik\ell}$

$$Be^{ik(\ell-1)} + Ae^{ik\ell} + Be^{ik(\ell+1)} = Ee^{ik\ell}$$

$$A + 2B\cos k = E$$

But any k? can only be N eigenvalues.

Q: What is resolution?

A: Boundary conditions

TD2

$$B\psi_{N-1} + A\psi_N + B\psi_1 = E\psi_N$$

$$A\psi_1 + B\psi_2 + B\psi_N = E\psi_1$$

If $\psi_N \equiv \psi_0$ then these eqns also are

$\psi_{N+1} \equiv \psi_0$

$A + 2B\cos k = E$

A Need $e^{ikN} = 1$ $\quad k = \frac{2\pi}{N}\{1, 2, \ldots N\}$

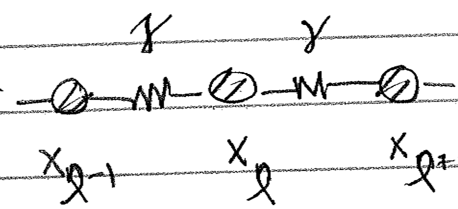
or $k = \frac{2\pi}{N}\left\{\frac{-N+1}{2}, \ldots \frac{N}{2}\right\}$ $\quad$ N k values $\Rightarrow$ N E values !

This matrix is encountered often enough it is

well worth discussing further.

1 Q: Has anyone encountered it before?

A: Coupled mass - spring system




$$m\ddot{x}_\ell = -\gamma(x_\ell - x_{\ell-1}) - \gamma(x_\ell - x_{\ell+1})$$

$$x_\ell(t) = \psi_\ell e^{i\omega t}$$

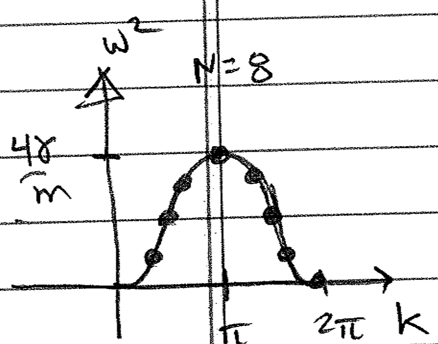
$$-m\omega^2\psi_\ell = -\gamma(\psi_\ell - \psi_{\ell-1}) - \gamma(\psi_\ell - \psi_{\ell+1})$$

TD3

$$m\omega^2 \psi_\ell = -\gamma \psi_{\ell-1} + 2\gamma \psi_\ell - \gamma \psi_{\ell+1}$$

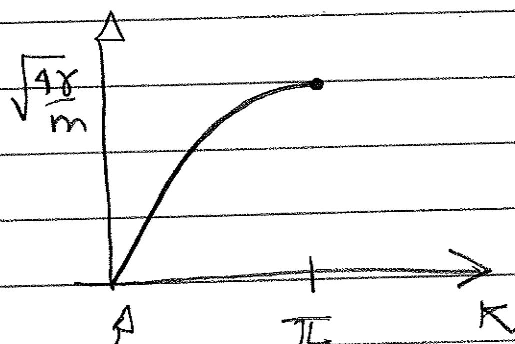
$$H = \begin{pmatrix} 2\gamma & -\gamma & 0 & 0 & \cdots \\ -\gamma & 2\gamma & -\gamma & 0 & \\ 0 & -\gamma & 2\gamma & -\gamma & \\ & & & & \end{pmatrix}$$

$$m\omega^2 = 2\gamma - 2\gamma \cos k$$




$$\omega^2 = \frac{2\gamma}{m}(1-\cos k) = \frac{4\gamma}{m} \sin^2 \frac{k}{2}$$

$$\omega = \sqrt{\frac{4\gamma}{m}} \sin \frac{k}{2}$$



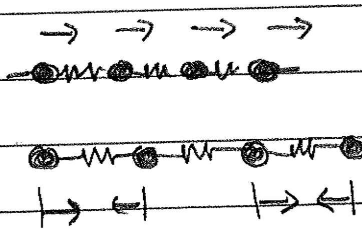

linear for
small k $\leftarrow$ Q? A phonons!

$|\psi\rangle \leftarrow$ normal mode eigenvector

$N=4 \qquad k = \frac{2\pi}{4}\{-1, 0, 1, 2\} = -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi$

$k=0 \qquad \psi_\ell = c e^{i 0 \ell} = c$

$k=\pi \qquad \psi_\ell = c e^{i\pi\ell} = c(-1)^\ell$



TD4

HW Q: looks like $k = -\pi/2$ $\quad \psi_\ell = e^{-i\pi/2\,\ell}$

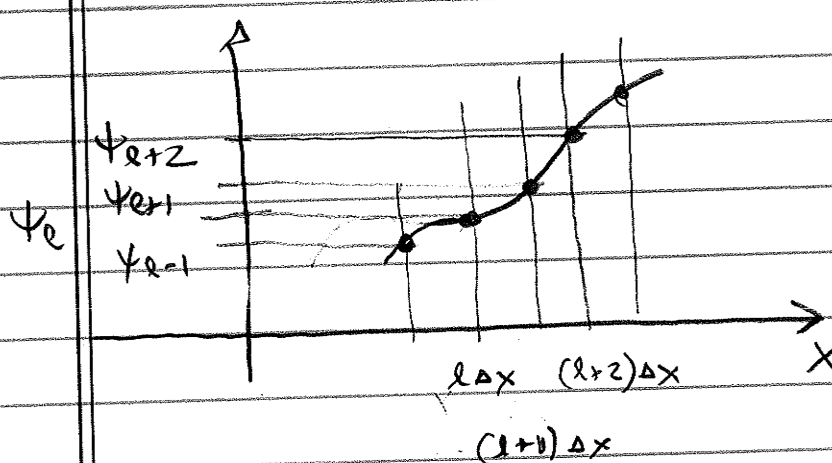
$k = \pi/2$ $\quad \psi_\ell = e^{+i\pi/2\,\ell}$

give complex $\psi_\ell \Rightarrow x_\ell(t)$ ?

What is resolution?

(2) $$\frac{d\psi}{dx} = \frac{\psi(x+\Delta x) - \psi(x)}{\Delta x}$$

$$\frac{d^2\psi}{dx^2} = Q? \qquad \cancel{\frac{\psi(x+\Delta x) - 2\psi(x) + \psi(x-\Delta x)}{(\Delta x)^2}}$$ HWQ




$$\frac{d^2\psi}{dx^2} = \frac{1}{\Delta x^2}\begin{pmatrix} 1 & -2 & 1 & & \\ & 1 & -2 & 1 & \\ & & 1 & -2 & 1 \end{pmatrix}\begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix}$$

This tri-diagonal matrix is discrete rep of second derivative operator

TD5

HW

(3) What is a good name for this operator:

$$T = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ & & & & \vdots \end{pmatrix}$$

Hint: What does $T$ do to $\begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_L \end{pmatrix}$

What do you notice about $T$ and any tridiagonal matrix (with PBC)

eigenvalues
What are eigenvectors of $T$?