PROBLEM SET 7 [Due Friday, March 22]

Physics 210– Computational Physics

The goal of this assignment is to introduce you to a problem which is almost of 'modern research level' which can be attacked with Monte Carlo. You are expected to try only *one* of the two choices.

[1.] This problem is known as ϕ^4 field theory. Consider a system with N degrees of freedom ϕ_i where $i = 1, 2, 3, \dots N$. Each ϕ_i is a real number between $-\infty$ and $+\infty$. Let $\vec{\phi} = (\phi_1, \phi_2, \dots, \phi_N)$. Using Monte Carlo, compute $\langle \phi_i^2 \rangle$ for the probability distribution,

$$p(\vec{\phi}) = c \; \exp[-\vec{\phi} A \vec{\phi} - \lambda \sum_{i=1}^{N} \phi_i^4].$$

Here A is the matrix,

$$A = \begin{pmatrix} 4 & 1 & 0 & \dots & 1 \\ 1 & 4 & 1 & \dots & 0 \\ 0 & 1 & 4 & \dots & 0 \\ \dots & \dots & \dots & \dots & \\ 1 & 0 & \dots & 1 & 4 \end{pmatrix}.$$

Do the problem for $N = 8$ and $\lambda = 0, 1, 2$.

[2.] This problem is known as the Ising Model. Consider a system with N degrees of freedom S_i where $i = 1, 2, 3, \dots N$. Each S_i is a discrete number which can take only the values -1 and $+1$. Using Monte Carlo, compute $\langle E \rangle$ for the probability distribution,

$$p(\{S_i\}) = c \; \exp[-E(\{S_i\})/T] \qquad\qquad E = -\sum_{i=1}^{N} S_i S_{i+1}$$

Do the simulations for $T = 0.0, 0.1, 0.2, \dots 4.0$ and make a plot of $\langle E \rangle$ versus T.