PROBLEM SET 3 [Due Thursday, January 31]

Physics 210– Computational Physics

The goals of this problem set are (i) to check a couple of random number generators; (ii) to use the random number generator to do a random walk problem; and (iii) to solve the 1-d Laplace equation numerically.

[**1.**] Code up the random number generator discussed in class.

Test the generator 'visually' by making a scatter plot using successive random numbers as the x and y coordinates of a point in two dimensions.

Compute the moments of the random number generator,

$$M_n = \langle r^n \rangle = \frac{1}{N} \sum_{i=1}^{N} r_i^n,$$

by generating N random numbers numerically. Check the moments have the desired values.

Compute the correlations of the random number generator

$$c(n) = \frac{1}{N-n} \sum_{i=1}^{N-n} (r_i - \tfrac{1}{2})(r_{i+n} - \tfrac{1}{2}).$$

Optional: Do the same for the random number generator ran2.c (ran2.f).

[**2.**] An experimental group manufactures a metallic surface consisting of an 21x21 square array of sites into which atoms can be adsorbed. A CO_2 molecule is placed at the center. It moves to a neighboring site to the right, left, up, and down randomly with probability $\frac{1}{4}$. What is the probability that it reaches the edge of the lattice within 100 steps?

[**3.**] Find the analytic solution of the equation,

$$-\frac{d^2\phi}{dx^2} = 6x,$$

with boundary conditions $\phi(0) = \phi(1) = 0$. What is the value of the quantity,

$$U = \int_0^1 [\ \tfrac{1}{2}(\frac{d\phi}{dx})^2 - 6x\phi\ ]\, dx\ ?$$

[**4.**] Write a program to solve the equation of problem one iteratively. Besides computing $\phi(x)$, also calculate U at each iteration. Let N be the number of mesh points on $(0, 1)$ and w be the relaxation parameter. Start with the solution $\phi(x) = 0$.

Which converges fastest, $w = \frac{1}{2}, 1$, or $\frac{3}{2}$?

How accurate is the solution for $N = 10, 20, 50$? How does the time to converge vary?