REVIEW STAT MECH

Bessel Function in Statistical Mechanics!

1-d chain of magnetic moments which can rotate in a plane

$$E = -K \sum_{\ell=1}^{N} \cos(\theta_\ell - \theta_{\ell+1})$$

ground state is $\theta_\ell = \text{const}$ $\quad E = -NJ$

"ferromagnetism"

$$Z = \sum_{\text{states}} e^{-\beta E}$$

$$= \int_0^{2\pi} d\theta_1 \int_0^{2\pi} d\theta_2 \ldots \int_0^{2\pi} d\theta_N \; e^{+\beta K \sum_{\ell=1}^{N} \cos(\theta_\ell - \theta_{\ell+1})}$$

$$= \quad '' \quad e^{\beta K \cos(\theta_1 - \theta_2)} \, e^{\beta K (\cos(\theta_2 - \theta_3)} \ldots$$

$$\int_0^{2\pi} d\theta_2 \, e^{\beta K \cos(\theta_1 - \theta_2)} \, e^{\beta K \cos \theta_2 - \theta_3)}$$

reminds us of what

$$M^2(\theta_1, \theta_3) = \int d\theta_2 \, M(\theta_1, \theta_2) \quad M(\theta_2, \theta_3)$$

$$Z = \int_0^{2\pi} d\theta_1 \int_0^{2\pi} d\theta_N \; M^{N-1}(\theta_1, \theta_N)$$

PBC $\quad K \cos(\theta_1 - \theta_N)$ term

In finite dim matrices + Hilbert space

~~Already encounter~~

$\langle x | p \rangle = e^{ipx/\hbar}$

$\langle x | \psi \rangle = \psi(x)$

$\langle x | p | \psi \rangle = \frac{\hbar}{i} \frac{\partial \psi}{\partial x}$

$\underbrace{\langle x | p | x' \rangle}_{\delta(x-x')\frac{\hbar}{i}\partial/\partial x'} \langle x' | \psi \rangle$

SM-2A

$$e^{\frac{x}{2}(t - 1/t)} = \sum J_n(x)\, t^n$$

How: $e^{ix\sin\theta} = \sum J_n(x) e^{in\theta}$

$$t = e^{i\theta}$$

$$e^{\frac{x}{2} 2\cos\theta} = \sum I_n(x)\, e^{in\theta}$$

$$e^{x\cos\theta} = \sum I_n(x)\, e^{in\theta}$$

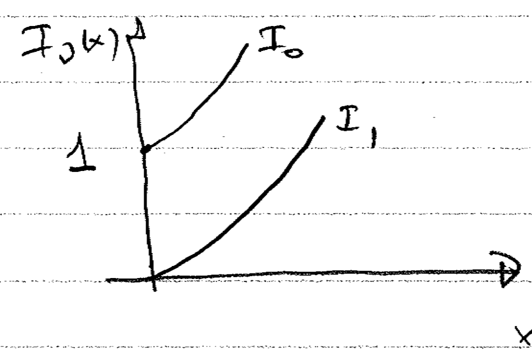
$J_n(x)$ enter $\nabla^2\psi = -k^2\psi$

$$e^{x/2\,(t + 1/t)} = \sum I_n(x)\, t^n$$

$I_n(x)$ enter $\nabla^2\psi = +k^2\psi$

$$J_\nu(x) = \sum_s \frac{1}{s!(s+\nu)!} (-1)^s \left(\frac{x}{2}\right)^{2s+\nu}$$

$$I_\nu(x) = \sum_s \frac{1}{s!(s+\nu)!} \left(\frac{x}{2}\right)^{2s+\nu}$$



$$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{2n!}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$= 1 - \frac{x^2}{2} + \frac{x^4}{24} - \ldots$$

another way $I_\nu(x) = J_\nu(ix)$ $\cosh x = \cos(ix)$

Trig analogues $e^{ix} = \cos x + i\sin x$ sinh x cosh

exponential sines/cosines

I_ν J_ν

$$Z = \int_0^{2\pi} d\theta_1 \quad M^N(\theta_1, \theta_1)$$

??

Trace M^N

Eigenvalues of $M \equiv \Lambda$

$$Z = \sum_\lambda \lambda^N = \lambda_{max}^N \sum_\lambda \left(\frac{\lambda}{\lambda_{max}}\right)^N$$

$$\to \lambda_{max}^N$$

Eigenvalues?

$$\int_0^{2\pi} M(\theta, \theta_1) f(\theta_1)\, d\theta_1 = \lambda f(\theta)$$

see page SM-2A!

$$e^{\beta K \cos(\theta - \theta_1)} = \sum_{n=-\infty}^{\infty} I_n(\beta K)\, e^{in(\theta-\theta_1)}$$

$$\sum_{n=-\infty}^{\infty} I_n(\beta K) \int_0^{2\pi} e^{in(\theta-\theta_1)} f(\theta_1)\, d\theta_1 = \lambda f(\theta)$$

$$\sum_{n=-\infty}^{\infty} I_n(\beta K)\, e^{in\theta} \int_0^{2\pi} e^{-in\theta_1} f(\theta_1)\, d\theta_1 = \lambda f(\theta)$$

$f(\theta) =$??

SM-3

$$f(\theta) = e^{im\theta} \quad !!$$

$$\int_0^{2\pi} e^{-in\theta_1} e^{im\theta_1}\, d\theta_1 = 2\pi\, \delta_{n,m}$$

$$I_n(\beta k)\, 2\pi\, e^{im\theta} = \lambda e^{im\theta}$$

$$\lambda = 2\pi I_n(\beta k)$$

$$I_n(x) = \sum \frac{1}{s!(s+n)!} \left(\frac{x}{2}\right)^{2s+n}$$

increasing n decreases $I_n(x)$

$$\therefore\ I_0(x) > I_1(x) > I_2(x) > \dots$$

$$\lambda_{max} = 2\pi I_0(\beta K)$$

$$\cancel{J_{1/2}(x) = \sum_s \frac{1}{s!(s+\frac{1}{2})!} (-1)^s \left(\frac{x}{2}\right)^{2s+1/2}}$$