

PHYSICS 204B, WINTER 2011
ASSIGNMENT SIX

Due Wednesday, March 16.

- [1.] Give two subgroups of $SU(2)$. Justify your answers.
- [2.] Consider the group S_3 which defines permutations of three objects a, b, c . Define its elements via $I(abc) = abc$, $G_{12}(abc) = bac$, $G_{13}(abc) = cba$, $G_{23}(abc) = acb$, $G_{123}(abc) = bca$, and $G_{312}(abc) = cab$.
- Is S_3 Abelian? Prove your answer.
 - What elements belong to the equivalence class of G_{12} ?
 - Is this equivalence class a subgroup? Prove your answer.
 - Give a subgroup of S_3 having at least two members.
- [3.] Show that there are eight generators of $SU(3)$.
- [4.] (I include this, even though we have done it in class, since it was on one of the past UCD Preliminary Exams). Show that the set of permutations of two elements is a group. What is its order? Write the multiplication table. How many classes are there? Is the group Abelian?
- [5.] Show that the set $\{1, -1, i, -i\}$ forms a group under algebraic multiplication. Write the multiplication table. How many classes are there?
- [6.] (UCD Preliminary Exam) Show that there are two groups of order four and determine their multiplication tables.
- [7.] Show that the set of 2x2 matrices of the form

$$\begin{pmatrix} x & y \\ -y & x \end{pmatrix}.$$

forms a group under (a) matrix addition, and (b) matrix multiplication.

- [8.] Show that the integers 1,2,3,4 form a group under multiplication mod 5. Write the multiplication table. What is the identity element? How many classes are there? Is the group Abelian?
- [9.] Show the transformations $x' = ax + b$ where x', x, a, b are real numbers, form a group as long as $a \neq 0$. What goes wrong for $a = 0$? Form a two dimensional representation of the group that acts on the vectors

$$\begin{pmatrix} x \\ 1 \end{pmatrix}.$$