PHYSICS 204B, WINTER 2011
ASSIGNMENT FIVE

Due Monday, February 28.

[**1.**] Show that

$$\Big(\frac{ia-1}{ia+1}\Big)^{ib} = \exp\big(-2b\cot^{-1}a\big)$$

for a and b real.

[**2.**] Find the analytic function $f(z)$ (a) if $u(x,y) = x^3 - 3xy^2$; and (b) if $v(x,y) = e^{-y}\sin x$.

[**3.**] Show that

$$\int_{(0,0)}^{(1,1)} z^* dz$$

depends on the path taken from $(0,0)$ to $(1,1)$ (a) by going first along the y-axis from $(0,0)$ to $(0,1)$ and then horizontally from $(0,1)$ to $(1,1)$; and (b) by going first along the x-axis from $(0,0)$ to $(1,0)$ and then vertically from $(1,0)$ to $(1,1)$.
Comment on the connection to whether $f(z)$ is analytic.

[**4.**] Show

$$\int_0^{\pi} \frac{d\theta}{(a+\cos\theta)^2} = \frac{\pi a}{(a^2-1)^{3/2}} \qquad \text{for } a > 1$$

[**5.**] Show

$$\int_{-\infty}^{\infty} \frac{x\sin x}{x^2+a^2}\, dx = \pi\, e^{-a}$$

[**6.**] Show

$$\int_0^{\infty} \frac{dx}{(x^2+a^2)^2}\, dx = \frac{\pi}{4a^3}$$

[**7.**] Evaluate

$$\int_{-\infty}^{\infty} \frac{x^2}{1+x^4}\, dx$$