Due Monday, February 28.

[1.] Show that
\[
\left( \frac{ia - 1}{ia + 1} \right)^b = \exp\left( -2b \cot^{-1}a \right)
\]
for \(a\) and \(b\) real.

[2.] Find the analytic function \(f(z)\) (a) if \(u(x, y) = x^3 - 3xy^2\); and (b) if \(v(x, y) = e^{-y} \sin x\).

[3.] Show that
\[
\int_{(0,0)}^{(1,1)} z^* dz
\]
depends on the path taken from \((0, 0)\) to \((1, 1)\) (a) by going first along the \(y\)-axis from \((0, 0)\) to \((0, 1)\) and then horizontally from \((0, 1)\) to \((1, 1)\); and (b) by going first along the \(x\)-axis from \((0, 0)\) to \((1, 0)\) and then vertically from \((1, 0)\) to \((1, 1)\).

Comment on the connection to whether \(f(z)\) is analytic.

[4.] Show
\[
\int_0^{\pi} \frac{d\theta}{(a + \cos \theta)^2} = \frac{\pi a}{(a^2 - 1)^{3/2}} \quad \text{for } a > 1
\]

[5.] Show
\[
\int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + a^2} dx = \pi e^{-a}
\]

[6.] Show
\[
\int_{0}^{\infty} \frac{dx}{(x^2 + a^2)^2} dx = \frac{\pi}{4a^3}
\]

[7.] Evaluate
\[
\int_{-\infty}^{\infty} \frac{x^2}{1 + x^4} dx
\]