Physics 204B, Winter 2011, Problem Set 3

[**1.**] By differentiating the generating function $g(t,x)$ with respect to t, multiplying by $2t$, and then adding $g(t,x)$, show that

$$\frac{1-t^2}{(1-2tx+t^2)^{3/2}} = \sum_{n=0}^{\infty}(2n+1)P_n(x)\,t^n$$

This result is useful in calculating the charge induced on a grounded metal sphere by a point charge q.

[**2.**] Verify the Dirac delta function expansions

$$\delta(1-x) = \sum_{n=0}^{\infty}\frac{2n+1}{2}P_n(x)$$

and

$$\delta(1+x) = \sum_{n=0}^{\infty}(-1)^n\frac{2n+1}{2}P_n(x)$$

These expressions appear in a resolution of the Rayleigh plane wave expansion into incoming and outgoing spherical waves. Note: Assume that the entire Dirac delta function is covered when integrating over $[-1,1]$.

[**3.**] Determine the electrostatic potential (Legendre expansion) of a circular ring of electric charge for $r < a$.

[**4.**] Calculate the electric field produced by a charged conducting ring for (a) $r > a$, and (b) $r < a$.

[**5.**] A uniformly charged spherical shell is rotating with constant angular velocity. (a) Calculate the magnetic induction $\vec{B}$ along the axis of rotation outside the sphere. (b) Using the vector potential series, find $\vec{A}$ and $\vec{B}$ for all space outside the sphere.

[**6.**] Verify by explicit calculation that

$$L_+Y_1^0(\theta,\phi) = -\sqrt{\frac{3}{4\pi}}\sin\theta\, e^{i\phi} = \sqrt{2}Y_1^1(\theta,\phi).$$

and

$$L_-Y_1^0(\theta,\phi) = +\sqrt{\frac{3}{4\pi}}\sin\theta\, e^{-i\phi} = \sqrt{2}Y_1^{-1}(\theta,\phi).$$