Physics 204B, Winter 2011, Problem Set 3

[1.] By differentiating the generating function \( g(t, x) \) with respect to \( t \), multiplying by \( 2t \), and then adding \( g(t, x) \), show that

\[
\frac{1 - t^2}{(1 - 2tx + t^2)^{3/2}} = \sum_{n=0}^{\infty} (2n + 1)P_n(x) t^n
\]

This result is useful in calculating the charge induced on a grounded metal sphere by a point charge \( q \).

[2.] Verify the Dirac delta function expansions

\[
\delta(1 - x) = \sum_{n=0}^{\infty} \frac{2n + 1}{2} P_n(x)
\]

and

\[
\delta(1 + x) = \sum_{n=0}^{\infty} (-1)^n \frac{2n + 1}{2} P_n(x)
\]

These expressions appear in a resolution of the Rayleigh plane wave expansion into incoming and outgoing spherical waves. Note: Assume that the entire Dirac delta function is covered when integrating over \([-1, 1]\).

[3.] Determine the electrostatic potential (Legendre expansion) of a circular ring of electric charge for \( r < a \).

[4.] Calculate the electric field produced by a charged conducting ring for (a) \( r > a \), and (b) \( r < a \).

[5.] A uniformly charged spherical shell is rotating with constant angular velocity. (a) Calculate the magnetic induction \( \mathbf{B} \) along the axis of rotation outside the sphere. (b) Using the vector potential series, find \( \mathbf{A} \) and \( \mathbf{B} \) for all space outside the sphere.

[6.] Verify by explicit calculation that

\[
L_+ Y_1^0(\theta, \phi) = -\sqrt{\frac{3}{4\pi}} \sin \theta \ e^{i\phi} = \sqrt{2} Y_1^1(\theta, \phi).
\]

and

\[
L_- Y_1^0(\theta, \phi) = +\sqrt{\frac{3}{4\pi}} \sin \theta \ e^{-i\phi} = \sqrt{2} Y_1^{-1}(\theta, \phi).
\]