Physics 204B, Winter 2011, Problem Set 1

[1.] In class we introduced the generating function of the Bessel polynomials,

$$g(x,t) = \exp\left[\frac{x}{2}\left(t - \frac{1}{t}\right)\right] = \sum_{n=-\infty}^{\infty} J_n(x) t^n \ .$$

By taking the derivative $\partial/\partial t$ on both expressions for g, derive the recurrence relation,

$$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x) \ .$$

[2.] Show the Laplace transform of the zeroth order Bessel function,

$$\mathcal{J}_0(a) = \int_0^\infty e^{-xa} J_0(bx)\, dx = 1 \,/\, \sqrt{a^2+b^2} \ .$$

Hint: One approach is to use the series expansion of $J_0(x)$. (I used a for the transform variable to avoid confusion with the summation variable s used in class for the series index.)

[3.] Since the trigonometric functions form a complete set (the idea behind Fourier series) we should be able to expand the Bessel functions in terms of them. Show that this is indeed this the case by proving

$$(a) \qquad \cos x = J_0(x) + 2\sum_{n=1}^{\infty}(-1)^n J_{2n}(x)$$

$$(b) \qquad \sin x = 2\sum_{n=1}^{\infty}(-1)^{n+1} J_{2n+1}(x)$$

Why would you expect the expansion of cos(sin) to involve only Bessel functions $J_n(x)$ of even(odd) order n?

[4.] Consider a cylinder of radius a and height h with potential $\psi = 0$ on all surfaces except for the top end section, which has a given potential $\psi(\rho, \phi)$. Write down an expression for the potential $\psi(\rho, \phi, z)$ everywhere inside the cylinder in terms of an expansion in the appropriate functions. Include an equation for the expansion coefficients. Can you evaluate the coefficients when $\psi(\rho, \phi) = \psi_0$, a constant?

[5.] Stone and Goldbart Problem 8-13.