Trig functions and sinh/cosh (Laplacian in Rectangular coordinates)

Before tackling Bessel, Legendre, ... Review how

$\sin x$, $\cos x$, $\sinh x$, $\cosh x$ arise in considering ∇^2

in rectangular coordinates.

We did a number of problems:

(1) $d = 1$ Diffusion Eqn, separated variables

$$D \frac{\partial^2 \psi}{\partial x^2} = \frac{\partial \psi}{\partial t} \qquad e^{ikx - Dk^2 t}$$

$\sin kx$
$\cos kx$

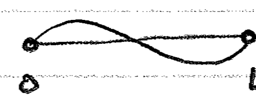
(2) $d = 1$ Wave eqn

$$\frac{\partial \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial \psi}{\partial t} \qquad f(x \pm vt)$$

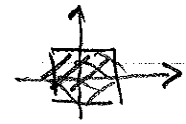
$\omega = vk$

But $\sin(kx \pm \omega t)$
$\cos(kx + \omega t)$

$k \to k_n$

when considering 

(3) $d = 2$ Diffusion Eqn (square cross section)



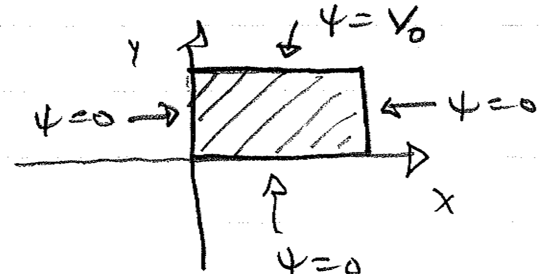
$$D\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \lambda\right)\psi = \frac{\partial \psi}{\partial t}$$

$\psi = 0$ on bdys

$\psi \sim \cos k_n x \cos k_m y \, e^{-\alpha t}$ $\alpha = -\lambda + D(k_n^2 + k_m^2)$

(4) $d = 2$ Laplace Eqn, (rectangular cross-section)

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi = 0$$



$\psi \sim \sin kx \; \underline{\underline{\sinh}}\, ky$

Much familiar about $\sin x$, $\cos x$ $\sinh x$, $\cosh x$

1) general functional form
2) series expansions
...

Less familiar, perhaps
1) orthogonality
2) completeness

} Fourier series, ...

GOAL: We want to develop same concepts and familiarity with Bessel, Legendre, etc.