[1.] Consider a damped harmonic oscillator subject to a “half-wave” driving force,

\[ F(t) = \sin \omega t \quad 0 < t < \pi / \omega \]
\[ F(t) = 0 \quad \pi / \omega < t < 2\pi / \omega . \]

where \( F(t) \) is periodic in \( t \) with period \( T = 2\pi / \omega \). Write down the long time form for \( x(t) \).

Just as in the sawtooth wave problem [2b] of assignment 4, plot the 2−, 4−, 6−, 8− term representations of the drive force \( F(t) \). Compare how well they do with the sawtooth case. Why is there a marked difference in convergence?

[2.] In class we derived a condition for a second order differential operator,

\[ \mathcal{L} = p_0(t) \frac{d^2}{dt^2} + p_1(t) \frac{d}{dt} + p_2(t) \]

to be Hermitian. As one example, \( \mathcal{L} = \frac{d^2}{dt^2} \) is Hermitian. Is the differential operator

\[ \mathcal{L} = m \frac{d^2}{dt^2} + \gamma \frac{d}{dt} + m\omega_0^2 \]

equenced in the damped oscillator problem Hermitian? What are its eigenfunctions? Are they complete?

[3.] Prove

\[ \frac{1}{L} \sum_{l=1}^{L} e^{2\pi i (n-m) t / L} = \delta_{nm} \]

[4.] Show that the general solution \( x(t) \) of a damped driven oscillator subject to an even force \( F(t) \) (i.e. only cosines in the Fourier expansion) of period \( T \) is given by,

\[ x(t) = \int_{0}^{T} dt' \mathcal{G}(t, t') F(t') \]
\[ \mathcal{G}(t, t') = \sum_{n} \frac{\cos \omega_n t' \cos (\omega_n t - \delta_n)}{(m^2(\omega_0^2 - \omega_n^2) + \gamma^2 \omega_n^2)^{1/2}} \]  
\[ \delta_n = \tan^{-1}(\gamma \omega_n/m(\omega_0^2 - \omega_n^2)) \]

with \( \omega_n = 2\pi n / T \). You may use all the equations we derived in class. The function \( \mathcal{G}(t, t') \) is called the Green’s function. This result is quite remarkable in the sense that it provides an expression for \( x(t) \) which is valid for any \( F(t) \). (This generality is the whole point of computing a Green’s function.)

[5.] Similarly, prove that the general solution \( \Psi(x, t) \) to a quantum problem with any initial wavefunction \( \Psi(x, 0) \) is,

\[ \Psi(x, t) = \int dx' \mathcal{G}(x, x', t) \Psi(x', 0) \]
\[ \mathcal{G}(x, x', t) = \sum_{n} \phi_n^*(x') \phi_n(x) e^{-iE_nt/\hbar} \]

where \( H\phi_n(x) = E_n\phi_n(x) \) are the eigenfunctions and eigenvalues of the time independent Schroedinger equation.