Physics 204A, Fall 2010, Problem Set 4

[1.] In class we defined the "length" $|f|$ of $f(x)$ to be $|f|^2 = \int f(x)^2 dx$ (for real functions). Stone and Goldbart comment that, more generally, one can define the length $|f|$ by $|f|^p = \int f(x)^p dx$, and that the triangle inequality is satisfied for $1 \leq p < \infty$. By considering $f(x) = 1$ and $g(x) = x$, show that the triangle inequality is indeed satisfied for $p = 2$ and $p = 3$, but is violated for $p = 1/2$. Use the interval $0 \leq x \leq 1$ for your domain for the functions.

[2a.] Calculate the Fourier series for the sawtooth wave, $f(x) = x$, for x in $(-\pi, \pi)$ and $f(x + 2\pi) = f(x)$. (Note that in class we defined $f(x)$ on the interval $(0, 2\pi)$ but it is fine to use any interval.) Make an argument based on the appearance of the function for why some of the a_n and b_n vanish.

[2b.] Calculate the sum of the Fourier series of [2a] using 4-, 6-, 8-, and 10- terms at $x/\pi = -1.00, -0.98, -0.96, \cdots, 0.00, 0.02, \ldots 1.00$. Plot your results.

[3.] Calculate the Fourier series for $f(x) = x^2$, for x in $(-\pi, \pi)$ and $f(x + 2\pi) = f(x)$. By plugging in $x = 0$ and $x = \pi$ evaluate the sums $S_1 = \sum_{n=1}^{\infty} 1/n^2$ and $S_2 = \sum_{n=1}^{\infty} (-1)^{n+1}/n^2$. By using Parseval's theorem (see Stone and Goldbart), evaluate $S_3 = \sum_{n=1}^{\infty} 1/n^4$.

[4.] In class we derived the Fourier series for the "square wave" function $f(x) = 1$ for $0 \leq x < \pi$ and $f(x) = 0$ for $\pi \leq x < 2\pi$. Write a program to evaluate the Fourier series for $x = \pi/8,\ \pi/4,\ 3\pi/2$. How many terms do you need to include to get the correct $f(x)$ to an accuracy of 0.04?