Physics 204A, Fall 2010, Problem Set 1

[1.] Stone and Goldbart, 1-3.

[2.] Stone and Goldbart, 1-4.

[3.] A ray of light starts from $(0, 1)$ and travels to $(2, 2)$ in a medium with index of refraction $n(x, y) = 1 + 0.1\,|y^2 - x^2|$.

a. Set up the functional J to be minimized so that the time to travel is least. Can you solve the Euler-Lagrange equation?

b. Restrict the paths to two straight line pieces with intermediate point (x_0, x_0). See Figure. Why might the time be low by traveling first to the $y = x$ line rather than directly from $(0, 1)$ to $(2, 2)$? Write down an expression for $t(x_0)$.

c. Write a program to evaluate $t(x_0)$ for the resticted paths of part (b). What is the minimum time? (With a bit of slightly painful algebra you can get the same answer analytically by setting $dt/dx_0 = 0$ in your answer to part (b). However, the point here is to practice some simple programming.)

d. How might you numerically determine the best path without the restriction to two straight line segments?

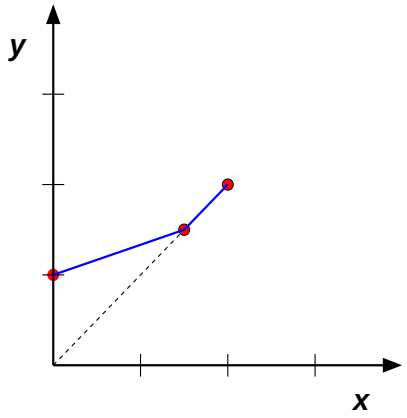

Figure 1: Starting position is at (0,1). Intermediate position is (x_0,x_0). Final position is at (2,2).