

MIDTERM EXAM, FALL 2005

Physics 204A– Mathematical Physics

[1.] (a) What are the eigenvalues and eigenvectors of

$$A = \begin{pmatrix} \pi/2 & 0 & 0 \\ 0 & \pi & \pi \\ 0 & \pi & \pi \end{pmatrix}?$$

(b) What matrix would you get if you diagonalized A ?

(c) What is the inverse of A ?

(d) Compute $\cos A$.

[2.] Prove that matrix multiplication is associative. That is, demonstrate $A(BC) = (AB)C$, where A, B , and C are $N \times N$ matrices.

[3.] (a) A small drop of ink is placed at the center of a glass of water and diffuses outward. How does the mean square displacement $\langle x^2 \rangle$ of the ink away from its original location depend on time t ? (Just consider the diffusion process. That is, ignore effects like a difference in buoyancy of ink and water.)

(b) Evaluate the sum S ,

$$S = \sum_{n=0}^N \binom{N}{n} x^n \quad \text{where} \quad \binom{N}{n} = \frac{N!}{n!(N-n)!}.$$

[4.] Obtain the Fourier expansions of the following functions:

$$\begin{aligned} f(x) &= -1 & -1 < x < 0 \\ f(x) &= 1 & 0 < x < 1 \\ f(x) &= 0 & \text{otherwise} \end{aligned}$$

and

$$\begin{aligned} g(x) &= -1 & -1 < x < 0 \\ g(x) &= 1 & 0 < x < 1 \\ &= \text{extended periodically (with period 2) from } -\infty \rightarrow +\infty \end{aligned}$$