MIDTERM EXAM

Physics 204A– Mathematical Physics

[1.] A and B are two matrices. What is the difference between $e^{it(A+B)}$ and $e^{itA}e^{itB}$ if t is small?

[2.] (a) What are the eigenvalues and eigenvectors of

$$A = \begin{pmatrix} 5 & 1 & 0 & 1 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 5 & 1 \\ 1 & 0 & 1 & 5 \end{pmatrix}?$$

You may use results we derived in class, or calculate directly.

(b) Suppose you apply the matrix A many times to the vector $\vec{v} = (.365 \;\; .104 \;\; .578 \;\; .722)$. In what direction will the resulting vector point? If you wanted the result to point in that direction to some desired degree of accuracy, what property of A would determine how big 'many' should be?

[3.] A string is clamped at both ends $x = 0$ and $x = L$. Assuming small amplitude vibrations, the amplitude $y(x,t)$ satisfies the wave equation,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}.$$

Here v is the wave velocity. The string is set in motion by grabbing it in the middle (at $x = L/2$) and displacing it so that it is in the shape of a triangle:

$$\begin{aligned} y(x,0) &= 2\,a\,x/L && 0 < x < L/2 \\ y(x,0) &= 2\,a\,(1 - x/L) && L/2 < x < L. \end{aligned}$$

The initial velocity is zero:

$$\frac{\partial y(x,t)}{\partial t} = 0.$$

Compute the subsequent displacement, $y(x,t)$.