FINAL EXAM, Fall 2005

Physics 204A– Mathematical Physics

[1.] What are the square roots of the complex number $1 + \sqrt{3}\, i$?

[2.] Is the function $f(z) = \mathrm{Re}[z] = x$ analytic? Explain your answer.

[3.] Find the analytic function $w(z) = u(x, y) + i\, v(x, y)$, if $u(x, y) = e^{-y} \sin x$.

[4.] (a) What is the Fourier transform of $f(x) = \delta(x - x_0)$?

(b) What is the Laplace transform of $f(t) = \delta(t - t_0)$?

(c) Solve

$$m \frac{d^2x}{dt^2} = P\, \delta(t)$$

for $x(t)$ assuming $x\,(t = -\infty) = 0$ and $v\,(t = -\infty) = 0$. (Note that physically $x(0) = v(0) = 0$ also, since the force is not applied until $t = 0$.) Make a plot of $x(t)$.

[5.] A random walk starts at $x = 0$ and consists of six steps. For each step, the probability of going to the right is $p = 2/5$ and of going to the left is $q = 3/5$. What are the possible final positions and what are their probabilities? What is the average distance from the origin? (A physical-reasoning based answer is fine. You don't have to try to do the binimial sums rigorously.)

[6.] The one dimensional diffusion equation with a δ-function source is

$$-D \frac{d^2\psi(x)}{dx^2} + A^2\, D\, \psi(x) = Q\, \delta(x)\ .$$

D, A, and Q are constants. Fourier transform this equation and compute $\phi(k)$. Write down an integral for $\psi(x)$. Can you do the integral? (Extra credit if you can, but don't waste too much time on it! We'll learn how to do this integral using complex variables at the beginning of next quarter.)

[7.] A triangular wave is represented by

$$\begin{aligned} f(x) &= x && 0 < x < \pi \\ f(x) &= -x && -\pi < x < 0\ . \end{aligned}$$

and is periodic with period 2π. Represent $f(x)$ by a Fourier series.

[8.] Show that the sum of the squares of the elements of a matrix remains invariant under orthogonal similarity transformations.