FINAL EXAM

Physics 204A– Mathematical Physics

[**1.**] Solve for $y(t)$ and $z(t)$ by Laplace transforming the equations.

$$\frac{dy}{dt} - y - 10z = 0 \qquad\qquad \frac{dz}{dt} + y + z = 0.$$

The initial conditions are $y(0) = 0$ and $z(0) = 2$.

[**2.**] Three equal masses m are connected by springs of spring constant k and $2k$ as shown in the figure. Write down the equations of motion. Assume a solution $x_l(t) = a_l e^{i\omega t}$ and convert the system of three differential equations into a set of three algebraic equations. What are the values of the normal mode frequencies ω? Give one of the eigenmodes. What is its interpretation?

[**3.**] (a) From the definition, derive the Laplace transform of $f(x) = x^2$.

(b) Find the inverse Laplace transform of $\frac{s}{(s+a)(s+b)}$ for $a \neq b$.

[**4.**] Compute the Fourier transform of the function which has period 2 and takes the values $f(x) = x$ for $-1 < x < 1$. Compute the Fourier transform of the function which takes the values $g(x) = x$ for $-1 < x < 1$, and is zero everywhere else. (See pictures.)

[**5.**] Compute the integral of $f(z) = x + 2yi$ on a circle of radius a about the origin $z = 0$. This function looks pretty well behaved in the sense that the real and imaginary parts are nice smooth polynomials. Shouldn't that mean the integral around a closed loop vanishes?

[**6.**] The functions $u(x, y)$ and $v(x, y)$ are the real and imaginary parts, respectively, of an analytic function $w(z)$.

(a) Assuming the required derivatives exist, show that $\nabla^2 u = \nabla^2 v = 0$.

(b) Show that the two curves $u(x, y) = c_1$ and $v(x, y) = c_2$ intersect at right angles.