FINAL EXAM

Physics 204A– Mathematical Physics

[**1.**] Verify that

$$\int_{(0,0)}^{(2,1)} z^* dz$$

depends on the path by evaluating the integral for the path which consists of moving horizontally from $(0,0)$ to $(2,0)$ and then vertically from $(2,0)$ to $(2,1)$, and the path which consists of moving vertically $(0,0)$ to $(0,1)$ and then horizontally from $(0,1)$ to $(2,1)$. By Cauchy's integral theorem, shouldn't the two paths give equal results? What went wrong?

[**2.**] Evaluate the integral

$$\int_C (z - z_0)^n dz$$

where C is a contour which consists of a circle of radius R about the point z_0.

[**3.**] From the definition of the Laplace transform, show that the transform of $f(x) = e^{-ax}$ is $F(s) = \frac{1}{s+a}$. Find the inverse Laplace transform of $\frac{1}{(s+a)(s+b)}$ for $a \neq b$.

[**4.**] The diffusion equation,

$$\frac{\partial \rho}{\partial t} = D \frac{\partial^2 \rho}{\partial x^2}$$

describes the evolution of the density $\rho(x,t)$. Suppose you discretize time in steps of dt and space into a mesh of size dx, so that $\rho(n,k)$ gives the density ρ at spatial location $x = n\,dx$ and time $t = k\,dt$. Derive an expression for $\rho(m, k+1)$ in terms of $\rho(n,k)$. That is, write the discretized version of the diffusion equation.

[**5.**] Prove that matrix multiplication is associative. That is, demonstrate $A(BC) = (AB)C$ if A, B, and C are NxN matrices.

[**6.**] A linear quantum oscillator in its ground state has a wavefunction

$$\psi(x) = a^{-1/2} \pi^{-1/4} e^{-x^2/2a^2}.$$

What is the corresponding momentum wave function $\phi(p)$?

BONUS QUESTION! $\psi(x)$ is normalized so that $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$. You notice that the momentum wave function is also normalized to unity. Is this an accident? Can you prove that for *any* normalized $\psi(x)$ the corresponding $\phi(p)$ will be normalized?