D1

Diffusion — Example of stochastic matrix

$$M = \begin{bmatrix} & \epsilon & & & \\ \epsilon & 1-2\epsilon & \epsilon & & \\ & \epsilon & 1-2\epsilon & \epsilon & \\ & & \epsilon & 1-2\epsilon & \epsilon \\ & & & \epsilon & \end{bmatrix}$$

Diagonal

$$\psi = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \leftarrow \text{"origin"}$$

$$\frac{\partial \psi}{\partial t} = D \frac{d^2\psi}{dx^2}$$

operator → matrix

$$\frac{\psi(x, t+\Delta t) - \psi(x,t)}{\Delta t} = D \frac{\psi(x+\Delta x, t) - 2\psi(x,t) + \psi(x-\Delta x, t)}{\Delta x^2}$$

$$M\psi = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \epsilon \\ 1-2\epsilon \\ \epsilon \\ 0 \\ \vdots \end{pmatrix} \qquad M^2\psi = \begin{pmatrix} \epsilon^2 \\ \epsilon(1-2\epsilon) + \epsilon(1-2\epsilon) \\ (1-2\epsilon)^2 + 2\epsilon^2 \\ \epsilon(1-2\epsilon) + \epsilon(1-2\epsilon) \\ \epsilon^2 \end{pmatrix}$$

$$M^2\psi = \begin{pmatrix} \epsilon^2 \\ 2\epsilon - 4\epsilon^2 \\ 1 - 4\epsilon + 6\epsilon^2 \\ 2\epsilon - 4\epsilon^2 \\ \epsilon^2 \end{pmatrix}$$

$$\psi(x, t+\Delta t) = \psi(x,t) + \frac{D\Delta t}{(\Delta x)^2} [\quad]$$

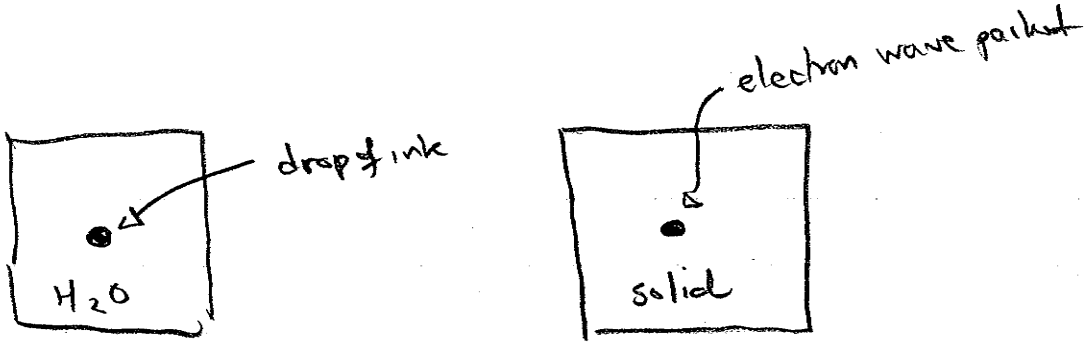
ϵ

what do you notice about $M\psi$ and $M^2\psi$?

Components add up to 1

"conservation of stuff"

D2



Relation between diffusion and random walks

$\epsilon = 1/2$

-4	0	0	0	0	1/16
-3	0	0	0	1/8	0
2	0	0	1/4	0	4/16
-1	0	1/2	0	3/8	0
origin	1	0	1/2	0	6/16
+1	0	1/2	0	3/8	0
+2	0	0	1/4	0	4/16
+3	0	0	0	1/8	0
+4	0	0	0	0	1/16

Binomial coefficients

Random Walk ≡ Diffusion with big diffusion constant (time step)

D1A

Eq $\epsilon = .1$

0	0	0	.001
0	0	.01	.024
0	.1	.16	.195
1	.8	.76	.560
0	.1	.16	.195
0	0	.01	.024
0	0	0	.001