$\boxed{1.}$ Infinite flat sheet of charge density σ in x-y plane.
oscillates along x-axis $\vec{V} = v_0 \hat{x} \cos\omega t$.

a) Find ~~all~~ all EM fields

b) Find power radiated / area a

The vector potential $\vec{A}$ obeys

$$(\nabla^2 - 1/c^2\, \partial^2/\partial t^2)\vec{A}(r,t) = \frac{4\pi}{c} J(r,t) \qquad \mu_0 J$$

with soln

$$\vec{A}(\vec{r},t) = \frac{1}{c}\int d\vec{r}'\, \vec{J}(r', t - |\vec{r}-\vec{r}'|/c) / |\vec{r}-\vec{r}'| \qquad \frac{1}{c} \to \frac{\mu_0}{4\pi}$$

if $\vec{J}(r',t') = \vec{J}(\vec{r}')e^{-i\omega t'}$ as in this cas

$$\vec{A}(\vec{r},t) = \frac{1}{c}\int d\vec{r}'\, \frac{\vec{J}(\vec{r}')\, e^{-i\omega t}}{|\vec{r}-\vec{r}'|} e^{i\omega/c|\vec{r}-\vec{r}'|}$$

$$\vec{A}(\vec{r},t) = \left\{\frac{1}{c}\int d\vec{r}'\, \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|} e^{ik|\vec{r}-\vec{r}'|}\right\} e^{-i\omega t} \qquad \frac{1}{c} \to \frac{\mu_0}{4\pi}$$

Units of J $= Q/At$

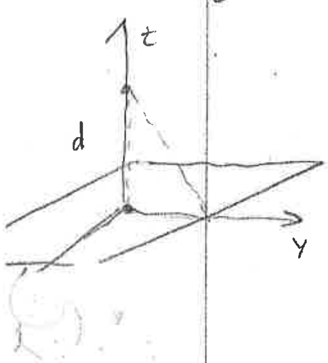
If we were looking at localized source and far field region, at this point we would start expanding $|\vec{r}-\vec{r}'|$ in powers of r'/r. However, here the source is clearly not localized and we better not do this.

Instead write $\vec{J}(r')$ explicitly and try to do integral

I believe $\vec{J}(\vec{r}) = \sigma V_0\, \delta(z)\hat{x}$ indep of x, y

$$\vec{A}(0,0,z,t) = \frac{\hat{x}}{c}\int dx'dy'dz'\, \frac{\sigma V_0 \delta(z')}{[x'^2+y'^2+(z'-z)^2]^{1/2}} e^{ik[x'^2+y'^2+(z'-z)^2]^{1/2}} e^{-i\omega t}$$

↑ clearly $\vec{A}$ indep of x, y by symmetry

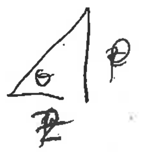


$$= \frac{\hat{x}}{c}\sigma V_0 \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy'\, \frac{e^{ik(x'^2+y'^2+z^2)^{1/2}}}{(x'^2+y'^2+z^2)^{1/2}} e^{-i\omega t}$$

change to plane polar coordinates $x' = \rho\cos\theta \quad y' = \rho\sin\theta$

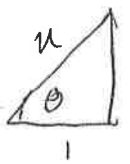
$$\vec{A}(x,y,z,t) = \frac{\hat{x}}{c}\,\sigma V_0 \int_0^\infty \int_0^{2\pi} d\theta\, \rho\, d\rho\ \frac{e^{ik\sqrt{\rho^2+z^2}}}{\sqrt{\rho^2+z^2}}\ e^{-i\omega t}$$

let $\rho = z\tan\theta$
$d\rho = z\sec^2\theta\, d\theta$



$$\vec{A}(x,y,z,t) = \frac{\hat{x}}{c}\,\sigma V_0\, 2\pi \int_0^{\pi/2} z\tan\theta\ z\sec^2\theta\, d\theta\ \frac{e^{ikz\sec\theta}}{z\sec\theta}\ e^{-i\omega t}$$

$$= \hat{x}\,\frac{2\pi\sigma V_0 z}{c}\, e^{-i\omega t} \int_0^{\pi/2} d\theta \sec\theta\tan\theta\ e^{ikz\sec\theta}$$



Now define $u = \sec\theta = 1/\cos\theta$
$du = \sec\theta\tan\theta\, d\theta$

$$\int_1^\infty du\ e^{ikzu}$$

$$= \lim_{\alpha\to 0} \int_1^\infty e^{-\alpha u + ikzu}\, du$$

$$= \lim_{\alpha\to 0} \frac{1}{-\alpha + ikz}\, e^{-\alpha u + ikzu}\Big|_1^\infty = \frac{e^{ikz}}{-ikz}$$

$$\vec{A}(x,y,z,t) = \hat{x}\,\frac{2\pi\sigma V_0 z}{c}\, e^{-i\omega t}\ \frac{i}{kz} e^{ikz}$$

$$\vec{A}(x,y,z,t) = \hat{x}\,\frac{2\pi\sigma V_0 i}{ck}\, e^{-i\omega t}\, e^{ikz}$$ ← take Real part $\sim \sin(kz-\omega t)$

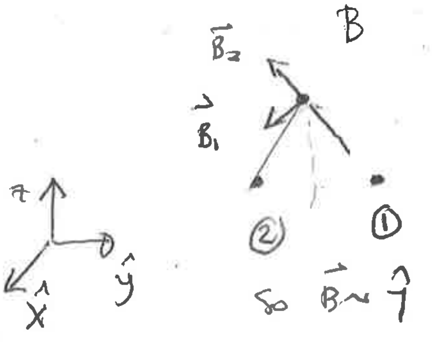
This is sort of expected $\vec{A}$ must be indep of x and y clearly by symmetry. Also with infinite plane there is nothing to set scale so expect $\vec{A}$ to be indep of z! Except omega has wavelength k.

$$\vec{B} = \nabla\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & 0 & 0 \end{vmatrix} = \hat{y}\,\frac{\partial A_x}{\partial z}$$

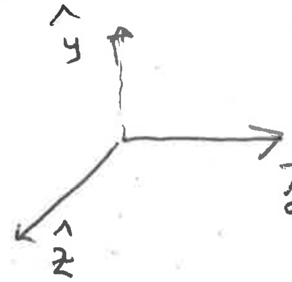
$$\vec{B} = -\hat{y}\,\frac{2\pi\sigma V_0}{c}\, e^{-i\omega t} e^{ikz}$$

$$\vec{B} = -\hat{y}\,\frac{\mu_0\sigma V_0}{2}\, e^{-i\omega t} e^{ikz}$$

$\sim \cos(kz-\omega t)$



so $\vec{B} \sim \hat{y}$



$\vec{J}$ in $\hat{x}$ direction

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[redacted]

Also $\phi(r,t) = \int d\bar{r}' \; \frac{\rho(r', t - \frac{|\bar{r}-\bar{r}'|}{c})}{|\bar{r}-\bar{r}'|} = \int dr' \; \frac{\sigma\,\delta(z')}{|x'^2+y'^2+(z'-z)^2|^{1/2}}$

$\rho(r',t) = \sigma\,\delta(z')$ $\qquad 4\pi \to 1/\epsilon_0$

$\phi(r,t) = \sigma \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \; \frac{1}{\sqrt{x'^2+y'^2+z^2}}$

$\vec{A} = \hat{x}\, \frac{\mu_0}{2} \frac{\sigma v_0 i}{k} e^{-i\omega t} e^{ikz}$

$\frac{d\phi}{dz} = +\sigma \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \; \frac{z}{(x'^2+y'^2+z^2)^{3/2}}$

$\frac{d\phi}{dz} = \sigma \int_0^{\infty} r\,dr \int_0^{2\pi} d\theta \; \frac{z}{(r^2+z^2)^{3/2}}$

$= \sigma\, 2\pi\, z \left. \frac{-1}{(r^2+z^2)^{1/2}} \right|_0^{\infty} = 2\pi\sigma$ as expected

$\to \sigma/2\epsilon_0$

$E =$

Thus $\bar{E} = -\frac{1}{c}\frac{\partial \bar{A}}{\partial t} + \bar{\nabla}\phi$

$$\boxed{\begin{aligned} \vec{E} &= 2\pi\sigma\hat{z} - \frac{2\pi\sigma v_0}{c}\hat{x}\, e^{-i\omega t} e^{ikz} \\ \vec{B} &= -\hat{y}\, \frac{2\pi\sigma v_0}{c} e^{-i\omega t} e^{ikz} \end{aligned}}$$

using $\omega = ck$

$4\pi \to 1/\epsilon_0$

$\frac{1}{c} \to \frac{\mu_0}{4\pi}$

b) $\vec{S} = \frac{c}{8\pi} \vec{E} \times \vec{B}^* = \frac{c}{8\pi} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ E_x & 0 & E_z \\ 0 & B_y^* & 0 \end{vmatrix} = \frac{c}{8\pi}\left[(-E_z B_y^*)\hat{x} + (E_x B_y^*)\hat{z}\right]$

time average $\to$ zero

$= \frac{c}{8\pi}\left(+2\pi\sigma \frac{2\pi\sigma v_0}{c}\hat{x} + \frac{2\pi\sigma v_0}{c}\frac{2\pi\sigma v_0}{c}\hat{z}\right)$

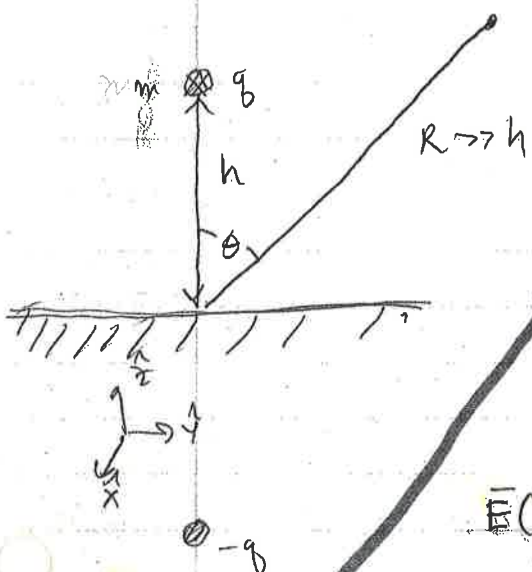
$\bar{S} = \frac{\pi}{2}\sigma^2 v_0 (\hat{x}) + \cancel{\hat{z}} + \frac{\pi}{2}\frac{1}{c}\sigma^2 v_0^2 \hat{z}$

$\vec{S}\cdot\hat{z} = \frac{\pi}{2}\sigma^2 v_0$

$\frac{\pi}{2c}\sigma^2 v_0^2$

Power radiated/area $= \frac{\pi}{2c}\sigma^2 v_0^2 \;\to\; \frac{1}{4\epsilon_0}\frac{1}{2}\frac{\mu_0}{4\pi}\sigma^2 v_0^2$

[4] Particle m, q dropped from rest height h above horizontal conductor plane. Calculate radiation power as a function of θ, R. Find polarization of radiation.



The charge q is accelerated with acceleration $-g\hat{z}$

A charge $-q$ (its image) is accelerated with acceleration $+g\hat{z}$ below plane

The radiation fields of an accelerated charge are given by

$$\vec{E}(\vec{r},t) = q\,\frac{\vec{r}\times\left[\left(\vec{r}-\frac{r\vec{v}}{c}\right)\times\dot{\vec{v}}\right]}{c^2\left(r-\frac{\vec{r}\cdot\vec{v}}{c}\right)^3}\Bigg|_{\tau = t - r/c}$$

$$\vec{B} = \frac{\vec{r}}{r}\times\vec{E} \qquad \text{where} \quad \vec{v} = \vec{v}(\tau),$$
$\vec{r} = \vec{r}(\tau)$ ← distance from position of particle at τ (retarded time) to observation point.

Note that when $v/c \ll 1$ this becomes

$$\vec{E}(\vec{r},t) = q\,\frac{\vec{r}\times(\vec{r}\times\dot{\vec{v}})}{c^2 r^3} \qquad \vec{B} = \frac{\vec{r}}{r}\times\vec{E}\ \frac{1}{c}$$

retarded τ implied in all this

We have $\dot{\vec{v}} = -g\hat{z}$ $\quad \vec{r} \approx R\sin\theta\,\hat{y} + R\cos\theta\,\hat{z}$ for $R \gg h$

so $\vec{r}\times\dot{\vec{v}} = -gR\sin\theta\,\hat{x}$ $\quad \vec{r}\times(\vec{r}\times\dot{\vec{v}}) = +gR^2\sin^2\theta\,\hat{z} - gR^2\cos\theta\sin\theta\,\hat{y}$

since $r \approx R$ we get $\vec{E}(\vec{r},t) \approx q\,\frac{g}{c^2R}\left(\hat{z}\sin\theta - \hat{y}\cos\theta\right)\sin\theta$

and $\vec{B} = q\,\frac{g}{c^2R}\left(\hat{x}\sin^2\theta + \hat{x}\cos^2\theta\right)\sin\theta = q\,\frac{g}{Rc^2}\,\hat{x}\sin\theta$ $\qquad \frac{1}{4\pi\epsilon_0}$

~~Then $\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{B} = \frac{c}{4\pi}\frac{g^2}{R^2c^4}\left(\hat{y}\sin\theta + \hat{z}\cos\theta\right)$~~ $\qquad \frac{1}{4\pi\epsilon_0 c^2} = \frac{\mu_0}{4\pi}$

$\vec{E} \perp \vec{r}$ expected?

~~$\vec{S} = \frac{1}{}\ \frac{g^2}{}\ \hat{n}$ independent of θ~~

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Must calculate image fields to this approximation all we have is that $q \Rightarrow -q$ and $g \Rightarrow -g$. This introduces identical contribution to both $\vec{B}$ and $\vec{E}$.

$$\vec{B}_T = \frac{2gq}{c^2 R}\hat{x}\sin\theta \qquad \vec{E}_T = \frac{2gq}{c^2 R}(\hat{z}\sin\theta - \hat{y}\cos\theta)\sin\theta$$

$$\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{B} = \frac{c}{\pi}\frac{g^2 q^2}{c^4 R^2}(\hat{y}\sin\theta + \hat{z}\cos\theta)\sin^2\theta$$

$\frac{1}{\mu_0}\vec{E}\times\vec{B}$

Note $|\vec{S}| \sim \frac{1}{R^2}\sin^2\theta$ gives dependence on θ and R

Dimensional check $\frac{q^2}{R} \sim$ energy $\frac{g^2}{c^3 R} \sim \frac{L^2}{T^4}\frac{T^3}{L^3 L} \sim \frac{1}{L^2 T}$

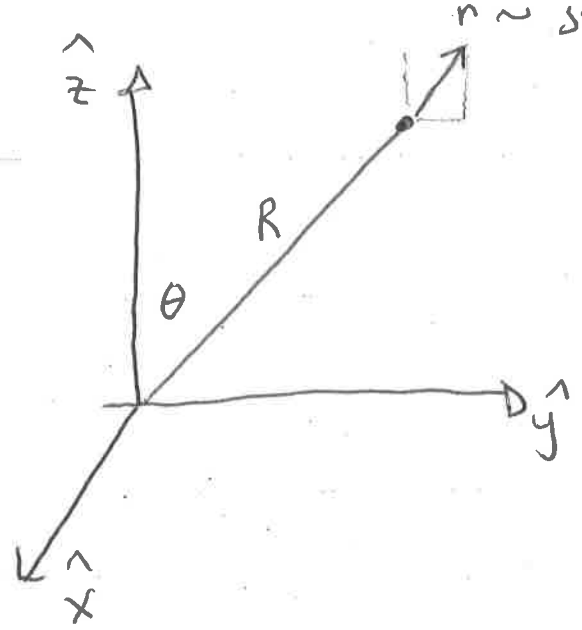
So $[S] \propto \frac{\text{energy}}{L^2 T}$ which is flux of energy as desired

The electric field is linearly polarized in the plane of $\hat{z}$ (i.e. particle motion) and $\vec{R}$. It is $\perp$ to $\vec{R}$ obviously

$\hat{r} \sim \sin\theta\,\hat{y} + \cos\theta\,\hat{z}$

radially outward

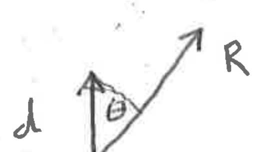
(1)



In general:

(2) $\vec{E} \sim \ddot{d}_\perp$

perpendicular component of dipole moment



so expect $\sin^2\theta$