1.

We will compute $\phi, \vec{A}$, then $\vec{E}, \vec{B}$

for point charge q following arbitrary path $\vec{r}_0(t)$

key new feature will be $\vec{E}, \vec{B} \sim 1/r$ pieces

which imply radiation. These result from acceleration

of charge.

$$\left.\begin{aligned} \rho(\vec{r},t) &= q\,\delta(\vec{r}-\vec{r}_0(t)) \\ \vec{j}(\vec{r}t) &= q\,\vec{v}_0\,\delta(\vec{r}-\vec{r}_0(t)) \end{aligned}\right\} \text{ charge and current density}$$

Scalar potential

$$\phi(r,t) = \phi_0(r,t) + \int d^3r' \int dt' \frac{\rho(r't')}{|r-r'|}\,\delta\left(t' - \left(t-\frac{R}{c}\right)\right)$$

observation point

$$\vec{R} = \vec{r} - \vec{r}'$$

observation — charge position

The only contribution is from when

$$t' = t - R/c = t - |\vec{r}-\vec{r}'|/c$$

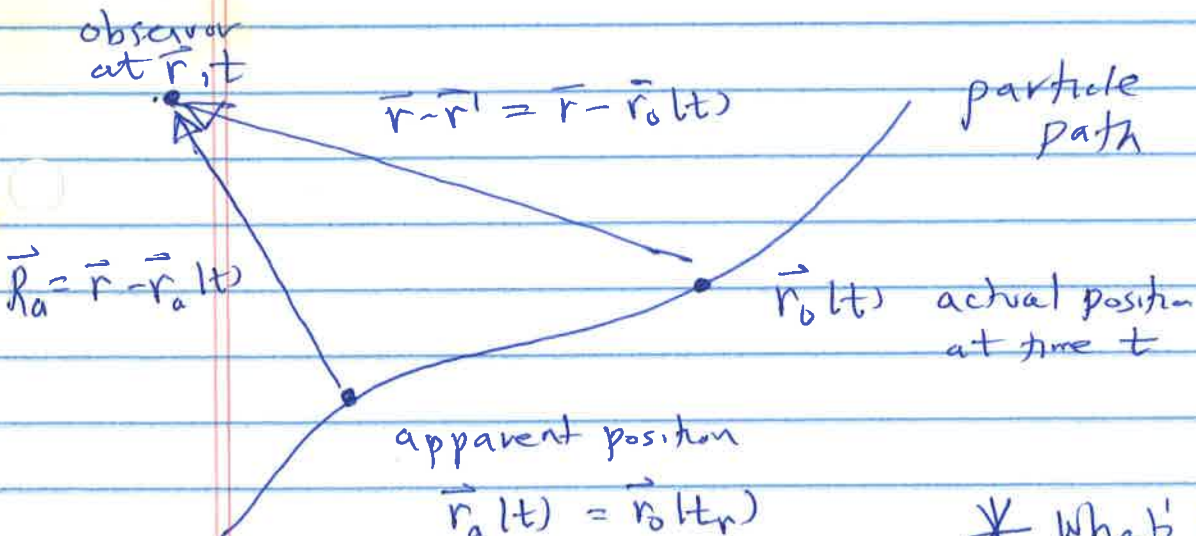
call this unique time $t' = t_r$ $\qquad t_r = t - \dfrac{|\vec{r}_0(t_r) - \vec{r}'|}{c}$

2. Write down * from page 6 and explain long calculation is all about second δ function!

In the case of uniform velocity (along $\hat{x}$ axis) the eqn relating t_r and t was possible to write and solve. Now it is harder and this is reason calculations are a bit complex.

(Rough) Analogy with scattering: if you know path explicitly computing scattering cross section is simple.

A picture helps a bit.



it is useful to be able to work both in terms of t and t_r. Hence these two different symbols for same position

* What's the trouble!? We want to do $\int dt'$ using $\delta(t' - (t - R/c))$. So we ~~need~~ require t' obeying this eqn! See comment on bottom of page 1

Give answers from page 7

Similar to two ways of expressing position,

it is useful to be able to think of velocity two ways

The observer sees

$$\vec{v}_r = \frac{d}{dt}\vec{r}_0(t)\Big|_{t=t_r} \qquad \vec{a}_r = \frac{d^2}{dt^2}\vec{r}_0(t)\Big|_{t=t_r}$$

Can use actual position at retarded time t_r

– or –

or apparent position at time t

$$\vec{v}_a(t) = \frac{d}{dt}\vec{r}_a(t) = -\frac{d}{dt}\vec{R}_a(t)$$

$$\vec{a}_a(t) = \frac{d^2}{dt^2}\vec{r}_a(t) = -\frac{d^2}{dt^2}\vec{R}_a(t)$$

recall
$\vec{R}_a(t) \equiv \vec{r} - \vec{r}_a(t)$
points from apparent position to observer

Consider a pulse emitted from t_r to $t_r + \Delta t_r$

It reaches observer at time $t_r + \frac{1}{c}R_a$

at this time the source has moved to $-\vec{R}_a + \vec{v}_r \Delta t_r$

so pulse terminates at

$$\underbrace{t_r + \Delta t_r}_{\text{time pulse ends}} + \frac{1}{c}\underbrace{|\vec{R}_a - \vec{v}_r \Delta t_r|}_{\text{added time to reach observer}}$$

$$t_r + \Delta t_r + 1/c\, |\vec{R}_a - \vec{v}_r \Delta t_r|$$

$$+ 1/c\, (R_a - \vec{v}_r \cdot \hat{R}_a \Delta t_r)$$

Just say how calculation goes

Since
$$|\vec{b} + \vec{db}|^2 = (\vec{b} + \vec{db}) \cdot (\vec{b} + \vec{db})$$
$$= b^2 + 2\vec{b} \cdot \vec{db} + \cancel{\vec{db}^2}$$
$$= b^2 (1 + 2\, \vec{db} \cdot \hat{b}/b)$$
$$|\vec{b} + \vec{db}| = b(1 + \tfrac{1}{2}\, 2\vec{db} \cdot \hat{b}/b)$$
$$= b + \vec{db} \cdot \hat{b}$$

to first order change in length involves only piece of $\vec{db}$ along $\hat{b}$

$$\rho \equiv \frac{1}{1 - \frac{\vec{v}_r}{c} \cdot \hat{R}_a}$$

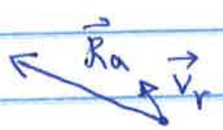
For observer pulse duration is

$$\Delta t = \Delta t_r \left(1 - \frac{\vec{v}_r}{c} \cdot \hat{R}_a\right) \equiv \Delta t_r / \rho$$

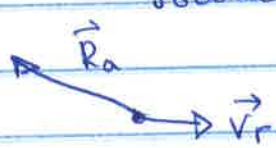
$\boxed{Q}$ What effect is this? Doppler Effect

Source moving towards observer $\Delta t < \Delta t_r$

$\vec{v}_r \cdot \hat{R}_a > 0$



Source moving away from observer $\Delta t > \Delta t_r$



Thus

$$d/dt = \gamma \, d/dt_r \qquad d/dt_r = 1/\gamma \, d/dt$$

and hence we can relate the two descriptions of velocity

$$\vec{V}_a = -\frac{d\vec{R}_a}{dt} = \frac{d\vec{r}_a}{dt} \qquad \vec{V}_a = \gamma \vec{V}_r$$

$\vec{r}_0(t_r) = \vec{r}_a(t)$

$$\vec{V}_r = \frac{d}{dt} r_0(t)\Big|_{t=t_r}$$

We can also relate how distance R_a between observer and apparent charge position changes with time

$$dR_a/dt = \hat{R}_a \cdot \frac{d\vec{R}_a}{dt} = -\hat{R}_a \cdot \vec{V}_a = -\gamma \hat{R}_a \cdot \vec{V}_r$$

$\vec{V}_a = \gamma \vec{V}_r$

Note:

$$\frac{db}{dt} = \frac{d}{dt}|\vec{b}| = \frac{d}{dt}\sqrt{\vec{b}\cdot\vec{b}}$$

$$= \frac{1}{2}\frac{1}{b}\, 2\vec{b}\cdot\frac{d\vec{b}}{dt}$$

$$= \hat{b}\cdot\frac{d\vec{b}}{dt}$$

6.

$$d R_a/dt = -\mathcal{J}\, \hat{R}_a \cdot \vec{v}_r \quad \text{with} \quad \begin{cases} \mathcal{J} = \dfrac{1}{1 - \frac{\vec{v}_r}{c}\cdot \hat{R}_a} \\ 1 - 1/\mathcal{J} = \frac{\vec{v}_r}{c}\cdot \hat{R}_a \end{cases}$$

Thus $$\frac{1}{c}\frac{d\bar{R}_a}{dt} = -\mathcal{J}(1 - 1/\mathcal{J})$$

$$= -\mathcal{J} + 1$$

$$\mathcal{J} = 1 - 1/c\; dR_a/dt \qquad \mathcal{J} = 1 - 1/c\; \mathcal{J}\, d R_a/dt_r$$

$$1/\mathcal{J} = 1 + \frac{1}{c}\, dR_a/dt_r \qquad \mathcal{J}\left(1 + 1/c\; dR_a/dt_r\right) = 1$$

Finally, the potential

$$\divideontimes \qquad \phi(rt) = q\int d^3r' \int dt' \frac{1}{|\bar{r}-\bar{r}'|}\, \underbrace{\delta(\bar{r}' - \bar{r}_0(t'))}_{\text{from } \rho(r')}\, \underbrace{\delta\!\left(t' - t + \frac{1}{c}|\bar{r} - \bar{r}_0(t')|\right)}_{\text{from G for wave eqn}}$$

Second delta function is satisfied at $t' = t_r$
(This was definition of t_r!)

Inside δ function is a function of t'. Use $\delta[f(x)] = \dfrac{1}{|f'(x_0)|}\,\delta(x - x_0)$

7.

$$\delta\left(t'-t+\frac{1}{c}|\vec{r}-\vec{r}_0(t')|\right) = \delta(t'-t_r)\cdot {}^1/_{\text{derivative}}$$

derivative of this wrt t'

$$1+\frac{1}{c}\frac{d}{dt'}|\vec{r}-\vec{r}_0(t')|$$

$$1+\frac{1}{c}\frac{d}{dt'}\sqrt{(\vec{r}-\vec{r}_0(t'))\cdot(\vec{r}-\vec{r}_0(t'))}$$

$$=1+\frac{1}{c}\frac{1}{|\vec{r}-\vec{r}_0(t')|}\,\frac{1}{2}\,(\vec{r}-\vec{r}_0(t'))\cdot 2\cdot\frac{d\vec{r}_0(t')}{dt'}(-1)$$

evaluate at $t'=t_r$ $\qquad \vec{R}_a \qquad \vec{v}_r$ because we evaluate at t_r

$$\text{derivative} = 1+\frac{1}{|\vec{r}-\vec{r}_0(t_r)|}\left(-\frac{\vec{v}_r}{c}\cdot\vec{R}_a\right) = {}^1/_{\varsigma}$$

So $\delta = \varsigma\,\delta(t-t_r)$

Finally $\phi(\vec{r},t) = q\int d^3r'\int dt'\,\frac{1}{|\vec{r}-\vec{r}\,'|}\,\delta(\vec{r}\,'-\vec{r}_0(t))\,\varsigma\,\delta(t'-t_r)$

Lienard-Wiechert

$$\boxed{\phi(\vec{r}\,t) = q\frac{\varsigma}{\underbrace{|\vec{r}-\vec{r}_0(t_r)|}_{R_a}} = \frac{q}{R_a-\frac{\vec{v}_r}{c}\cdot\vec{R}_a}}$$

Similar calculation

$$\vec{A}(\vec{r}\,t) = q\frac{\varsigma\,\vec{v}_r/c}{R_a} = \frac{q\,\vec{v}_r/c}{R_a-\vec{v}_r/c\cdot\vec{R}_a}$$