LWF 1

$\boxed{\text{The Fields}}$

$= 1 - \vec{v}_r/c \cdot \vec{R}_a$

We obtained the potentials $\quad \varsigma = 1 - 1/c \; dR_a/dt$

$$\phi(rt) = q\varsigma/R_a = q/(R_a - \vec{v}_r/c \cdot R_a) \qquad q/4\pi\epsilon_0$$

$$\vec{A}(rt) = q\varsigma\frac{\vec{v}_r}{cR_a} = q\,\vec{v}_r/c \Big/ (R_a - \vec{v}_r/c \cdot \vec{R}_a) \qquad \frac{\mu_0}{4\pi} q$$

These are in <u>Gaussian</u> units.

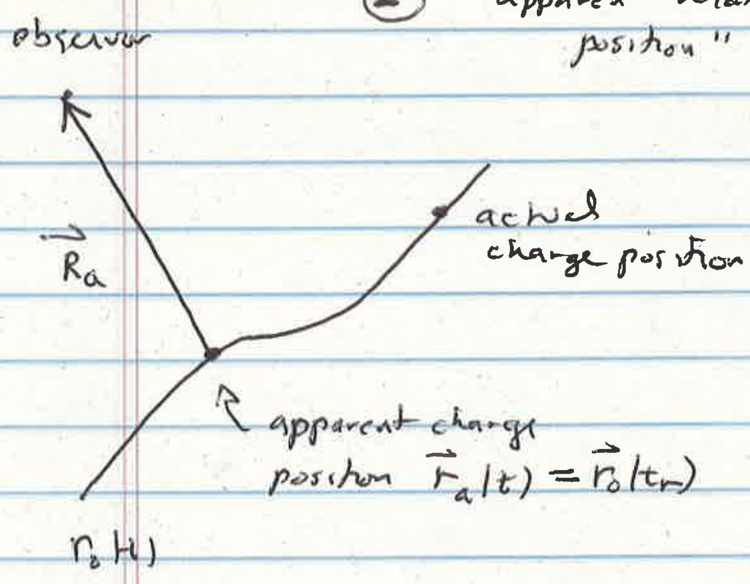
Recall the definitions $\quad \vec{R}_a(t) = \vec{r} - \vec{r}_0(t_r) = \vec{r} - \vec{r}_a(t)$

vector from source to observer

retarded time

"apparent position"

$\ominus$ "apparent relative position"



R_a

LWF 2

There are 3 pieces to $\vec{E}$. The first two, $\vec{E}_1$ and $\vec{E}_2$, originate in $\vec{\nabla}\phi$ while the third, $\vec{E}_3$, is from $\partial\vec{A}/\partial t$

$$\phi(\vec{r}\,t) = q\int d^3r'\int dt'\,\frac{1}{|\vec{r}-\vec{r}'|}\,\delta(\vec{r}-\vec{r}_0(t'))\,\delta(t-t'-1/_c R)$$

If $\vec{\nabla}\phi$ acts on $1/R = 1/|\vec{r}-\vec{r}'|$ we get $\hat{R}/R^2$ so

$$\vec{E}_1(\vec{r},t) = q\int d^3r'\int dt'\,\frac{\hat{R}}{R^2}\,\delta(\vec{r}-\vec{r}_0(t'))\,\delta(t-t'-1/_c R)$$

But $\vec{\nabla}\phi$ can also act on the time delta function. Using the chain rule

$$\frac{\partial}{\partial x}\delta(t'-t+1/_c \underbrace{R}_{|\vec{r}-\vec{r}'|}) = \delta'(t'-t+\tfrac{1}{c}R)\,\frac{1}{c}\,\frac{(x-x')}{R}$$ ← x component of $\vec{R} = R\hat{R}$

Therefore

$$\vec{E}_2(\vec{r}\,t) = \frac{q}{c}\int d^3r'\int dt'\,\frac{\hat{R}}{R}\,\delta(\vec{r}-\vec{r}_0(t'))\,\delta'(t-t'-1/_c R)$$

$$= q/_c \frac{d}{dt}\int d^3r'\int dt'\,\frac{\hat{R}}{R}\,\delta(r-r_0(t'))\,\delta(t-t'-1/_c R)$$

Finally the $-\frac{1}{c}d\vec{A}/dt$ term,

$\frac{\mu_0 q}{4\pi}$
$$\vec{A} = q/_c\int d^3r'\int dt'\,\frac{\vec{v}_0(t')}{|\vec{r}-\vec{r}'|}\,\delta(\vec{r}'-\vec{r}_0(t'))\,\delta(t-t'-\tfrac{1}{c}R)$$

$$-\frac{1}{c}\frac{d\vec{A}}{dt} = \underbrace{-q/_{c^2}}_{\frac{\mu_0 q}{4\pi}}\,d/_{dt}\int d^3r'\int dt'\,\frac{\vec{v}_0(t')}{R}\,\delta(\vec{r}'-\vec{r}_0(t'))\,\delta(t-t'-1/_c R)$$

LWF3

If we put $\vec{E}_2$ and $\vec{E}_3$ together we obtain

$$\vec{E}_{23} = q\frac{1}{c}\frac{d}{dt}\int d^3r'\int dt' \; \frac{\hat{R} - \vec{v}(t')/c}{R}\,\delta(\vec{r}' - \vec{r}_0(t'))\,\delta(t - t' - R/c)$$

$$= q\frac{1}{c}\frac{d}{dt}\left[\frac{\rho}{R_a}(\hat{R}_a - \vec{v}_r/c)\right]$$

This last step follows from mathematics identical to the

find integrations ~~when~~ over $\int d^3r'\int dt'$ in obtaining ϕ, the

key step of which was $\delta(t - t' - R/c) = \rho\,\delta(t' - t_r)$

We want to understand how many time derivatives there

are in the different pieces of $\vec{E}$. $\vec{E}_1$ gives the only piece

with zero time derivatives $\vec{E}_1 = q\rho\hat{R}_a/R_a^2$ with $\rho = 1 - \frac{1}{c}dR_a/dt$

$$\vec{E}^{(0)} = q\hat{R}_a/R^2$$

The piece with one time derivative comes from $\vec{E}_1$ and part of the $\rho\hat{R}_a/R_a$

piece of $\vec{E}_{23}$

$$\vec{E}^{(1)} = -q\frac{1}{c}\frac{\hat{R}_a}{R_a^2}\frac{dR_a}{dt} + \frac{q}{c}\frac{d}{dt}\hat{R}_a/R_a$$

$$= q\,R_a/c\;d/dt\left(\hat{R}_a/R_a^2\right)$$

Since $\frac{d}{dt}\hat{R}_a/R_a = \frac{d}{dt}\left(\frac{\hat{R}_a}{R_a^2}R_a\right) = \frac{d}{dt}\left(\frac{\hat{R}_a}{R_a^2}\right)R_a + \frac{\hat{R}_a}{R_a^2}\frac{dR_a}{dt}$

LWF 4

The term with two time derivatives comes from the other part of the $\frac{\hat{R}_a}{R_a}$ term in $\vec{E}_{23}$ and the V_r/c term in $\vec{E}_{23}$

$$\vec{E}^{(2)} = q/c^2 \, d/dt \left[\frac{-\hat{R}_a}{R_a} \frac{dR_a}{dt} + \frac{1}{R_a} \frac{d\vec{R}_a}{dt} \right]$$

from $1/c \, dR_a/dt$ piece of $\mathcal{J}$

$$\frac{V_r}{c} = \frac{1}{c} \frac{dr_0}{dt}\Big|_{t_r} = \frac{-1}{c} \frac{d\vec{R}_a}{dt}\Big|_{t_r}$$

since $R_a^{(t)} = r - r_0(t_r)$

As for $\vec{E}^{(1)}$, the two terms can be combined using the product rule

$$\frac{d\hat{R}_a}{dt} = \frac{d}{dt} \frac{1}{R_a} \vec{R}_a$$

$$= \underbrace{-\frac{1}{R_a^2} \frac{dR_a}{dt} \vec{R}_a}_{-\frac{1}{R_a} \frac{dR_a}{dt} \hat{R}_a} + \frac{1}{R_a} \frac{dR_a}{dt}$$

$$\vec{E}^{(2)} = q/c^2 \, d/dt \, d\hat{R}_a/dt$$

All together

first derived by Feynman

Add $\frac{1}{4\pi\epsilon_0}$

$$\boxed{\vec{E} = q \frac{\hat{R}_a}{R_a^2} + q \frac{R_a}{c} \frac{d}{dt} \frac{\hat{R}_a}{R_a^2} + q \frac{1}{c^2} \frac{d^2 \hat{R}_a}{dt^2}}$$

Heaviside - Feynman formulae for fields

Similar sorts of algebra

$$\boxed{\vec{B} = q \frac{\vec{V}_a \times \hat{R}_a}{c R_a^2} + q \frac{d}{dt} \frac{\vec{V}_a \times \hat{R}_a}{c^2 R_a} = \hat{R}_a \times \vec{E}}$$

$\frac{1}{c} \to \frac{\mu_0}{4\pi}$

due to Heaviside

$1/c \; \hat{R}_a \times \vec{E}$

LWF5

Physical interpretation of Feynman formulae

$$\vec{E} = q\frac{\hat{R}_a}{R_a^2} + q\frac{R_a}{c}\frac{d}{dt}\frac{\hat{R}_a}{R_a^2} + q\frac{1}{c^2}\frac{d^2}{dt^2}\hat{R}_a$$

Coulomb field due to q at apparent (retarded) position

time derivative of field times delay so partially "corrects" first term

radiation field we will show $\sim 1/R$ Dominant at large R Has some $1/R^2$ pieces

very close to instantaneous Coulomb field, ie Coulomb field evaluated at actual position

There is also a "Lienard-Weichert" form for $\vec{E}$ and $\vec{B}$ which separates $\vec{E}$ into $1/R$ and $1/R^2$

$$\vec{E} = \frac{q(1-v_r^2/c^2)}{\left(R_a - \frac{\vec{v}_r}{c}\cdot\vec{R}_a\right)^3}\left(\vec{R}_a - \frac{\vec{v}_r}{c}R_a\right)$$

only term if $\dot{\vec{v}}_r = 0$ (no acceleration) compare to constant $\vec{v}$ calculation of last week

$\frac{1}{4\pi\epsilon_0}$

$$+ \frac{q}{c^2\left(R_a - \frac{\vec{v}_r}{c}\cdot\vec{R}_a\right)^3}\left(\dot{\vec{v}}_r \times \left(\vec{R}_a - \frac{\vec{v}_c}{c}R_a\right)\right)\times\vec{R}_a$$

$$\vec{B} = \hat{R}_a \times \vec{E}$$

$1/c$

$R_a - \frac{\vec{v}_r}{c}\cdot\vec{R}_a \to 0$ if $\vec{v}_r \parallel \vec{R}_a$ and $v_r \to c$

synchrotron radiation concentrated in direction of motion