R1.

Relativity

The laws of classical mechanics are the same in all "inertial reference frames" ← at rest or moving with constant velocity

How defined: Newton's First law holds

object experiencing no forces moves in straight line at constant speed

Mathematically $\vec{F} = m\vec{a} = m\frac{d^2\vec{r}}{dt^2}$

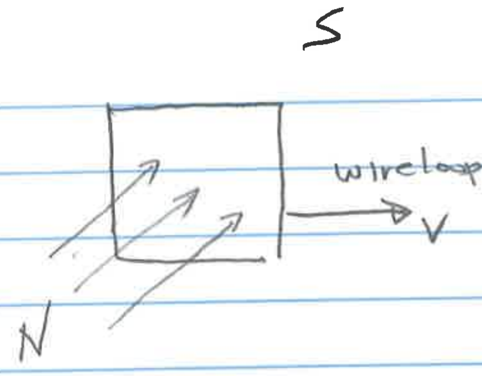
$$\vec{r}\,' = \vec{r} + \vec{v}t$$

$$\vec{a}\,' = \frac{d^2\vec{r}\,'}{dt^2} = \vec{a}$$

but do forces depend on $\vec{v}$? No, even friction is relative velocity of two objects in the frame

E+M appears to violate relativity. In rest frame of charge it produces $\vec{E}$ but not $\vec{B}$. Likewise Lorentz force depends on $\vec{v}$.

R2



1) To observor moving with wire loop: The loop is at rest. There are no magnetic forces on charges in loop. But a magnet moves past, changing flux produce electric field (Faraday). Electric force gives Emf $-d\Phi/dt$

2) observor at rest wrt magnets: The wire loop is in motion. charges feel a magnetic force. If one computes the motional EMF will get $\mathcal{E} = -d\Phi/dt$

$$\vec{F}_{mag} = q\,\vec{v}\times\vec{B} \to qvB$$

$$F\ell = qB\underbrace{v\ell}_{dA/dt} \to q\frac{d\Phi}{dt}$$

Same result!

This suggest Electromagnetic phenomena also might obey principle of relativity.

R3

Of course relativity tied to issue of presence of special inertial reference frame at rest with respect to the "ether" the medium whose mechanical vibrations carry light

Michelson-Morley interpreted expt as confirmation of "etherdrag"

Michelson-Morley expt: speed of light is same in all directions even though earth moving at 50 km/s around sun, and sun around galaxy, and galaxy in universe

$5 \cdot 10^4$ m/s

$\sim 10^{-4} c$

Lorentz-Fitzgerald ether wind compresses matter to conceal the speed variation

SIMILAR IDEA TO LIGO

280 trips of 4Km arm!

R4

Einstein:

- Laws of physics same in all inertial frames

- speed of light in vacuum is the same for all inertial observers

requires replacing $V_{AC} = V_{AB} + V_{BC}$

by $$V_{AC} = \frac{V_{AB} + V_{BC}}{1 + V_{AB} V_{BC}/c^2}$$

reduces to "common sense" rule when $V_{AB}, V_{BC} \lll c$

and when $V_{BC} = c$

$$V_{AC} = \frac{V_{AB} + c}{1 + V_{AB}/c} = c$$

Relativity is governed mathematically by Lorentz transformations. Before introducing these, consider 3 key phenomena qualitatively.

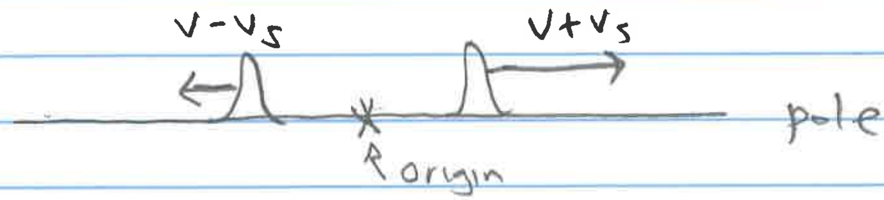
R5A

Sound problem cont'd

Observer outside train

Sound going to right moves at $v+v_s$

Sound going to left moves at $v-v_s$



pulse traveling to right: $(v+v_s)t_B = \ell/2 + vt_B$

pulse traveling to left: $(v-v_s)t_A = -\frac{\ell}{2} + vt_A$

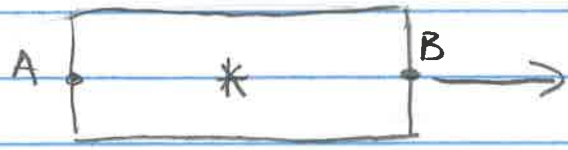
$$\left.\begin{aligned} t_B &= \ell/2v_s \\ t_A &= \ell/2v_s \end{aligned}\right\}$$ Same answer as observer on train

and <u>simultaneous</u>

R5

Qualitative Discussion of Relativity

* Relativity of simultaneity



Moving train car
light emitted from center

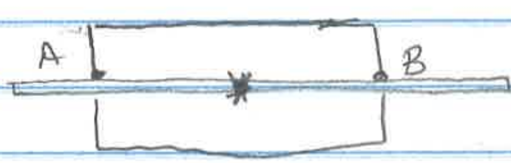
1) observer inside train : light reaches A and B at same time

2) observe on ground : light reaches A before B

Events which are simultaneous in one frame are not so in another

Important : Discussion has nothing to do with time it takes for result of expt to get to observer. The observer has assistants at A and B with synchronized watches who recorde arrival of light right at the scene.

What would happen to sound waves



← freely sliding pole

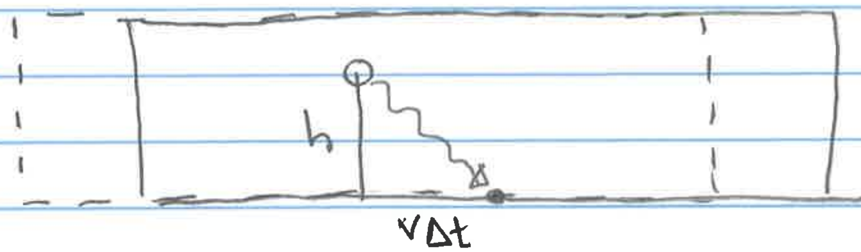
* strike pole at center so sound wave propagates

What does observer outside train say?

R6

* Time Dilation

Consider light ray leaving bulb going downwards



Observor on train $\Delta t_0' = h/c$

observor outside train $\Delta t' = \sqrt{h^2 + v^2(\Delta t)^2}\,/c$

$$c^2(\Delta t)^2 = h^2 + v^2(\Delta t)^2$$

Again interesting to analyze this for sound!

$$\Delta t = \sqrt{\frac{h^2}{c^2-v^2}} = \frac{h}{c}\frac{1}{\sqrt{1-v^2/c^2}}$$

Assumes "h" is same for two observors see page R9

$$\Delta t = \Delta t_0' \, 1/\sqrt{1-v^2/c^2} = \Delta t_0' \gamma$$

$$\Delta t_0' = \sqrt{1-v^2/c^2}\,\Delta t$$

Moving clocks run slow < 1

Evidence: muon lifetime $= 2 \cdot 10^{-6}$ sec

particle accelerators muons moving close to speed of light: lifetime $\gg 2 \cdot 10^{-6}$

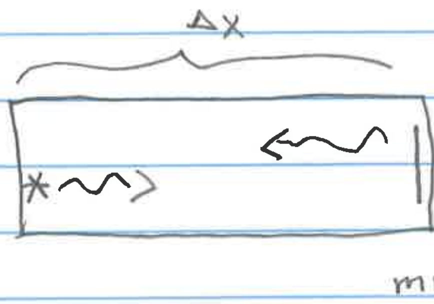
R7

Possible contradiction: Who is moving, observer on train or observer on ground? Then if this is not answerable, whose clock runs slow?

Two observers, each with cargo of muons. They move with velocity v wrt each other. Who has more muons in the future? Answer tied to question of when in future they compare: Simultaneous does not have same meaning for them, and they need to compare at the same time.

R8

* Lorentz contraction



Observer in train

$$\Delta t' = \frac{2\Delta x'}{c}$$

observer on ground

$$\Delta t_1 = \frac{\Delta x' + v\Delta t_1}{c}$$ ← mirror moving away light travels extra distance

$$\Delta t_2 = \frac{\Delta x' - v\Delta t_2}{c}$$ ← receiver moving towards. light travels less far

$$\Delta t_1' = \frac{\Delta x'}{c - v}$$

$$\Delta t_2' = \frac{\Delta x'}{c + v}$$

$$\Rightarrow \Delta t = \Delta x' \frac{2c}{c^2 - v^2}$$

$$= \frac{2\Delta x'}{c} \frac{1}{1 - v^2/c^2}$$

But $\Delta t' = \sqrt{1 - v^2/c^2}\, \Delta t$

So $$\frac{2\Delta x'}{c} = \sqrt{1 - \frac{v^2}{c^2}} \frac{2\Delta x'}{c} \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\Delta x' = \frac{\Delta x}{\sqrt{1 - v^2/c^2}}$$ ← Observer on ground

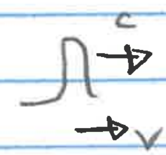
observer on train

$\boxed{\text{Moving objects are shortened}}$

$\Delta x = \sqrt{1 - \frac{v^2}{c^2}}\, \Delta x' < \Delta x'$

R-8A

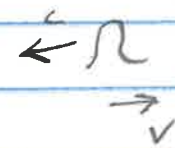
Again, amusing to check for Lorentz contraction via "usual" or "classical" notion of velocity addition



$$\Delta t_1 = \frac{\Delta x + v\Delta t_1}{c+v}$$

speed of light is changed by motion of train

$$\Delta t_2 = \frac{\Delta x - v\Delta t_1}{c-v}$$



Evidently

$$c\Delta t_1 = \Delta x$$

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{2\Delta x}{c}$$

$$c\Delta t_2 = \Delta x$$

agrees perfectly with

$$\Delta t' = \frac{2\Delta x'}{c}$$

R9

Ladder in barn paradox, like bundle of muon paradox, has resolution in question of simultaneity:

"In the barn" means both ends of ladder inside at same time, and 2 observers disagree about what "at same time" means.

Lorentz contraction applies only ~~to~~ along direction of motion, Dimensions $\perp$ to velocity are not contracted.

Wall beside railroad tracks
passenger on train holds red paint brush out of window 1 meter above train floor in her frame.
observer outside train paints blue line 1 m above ground in her frame.

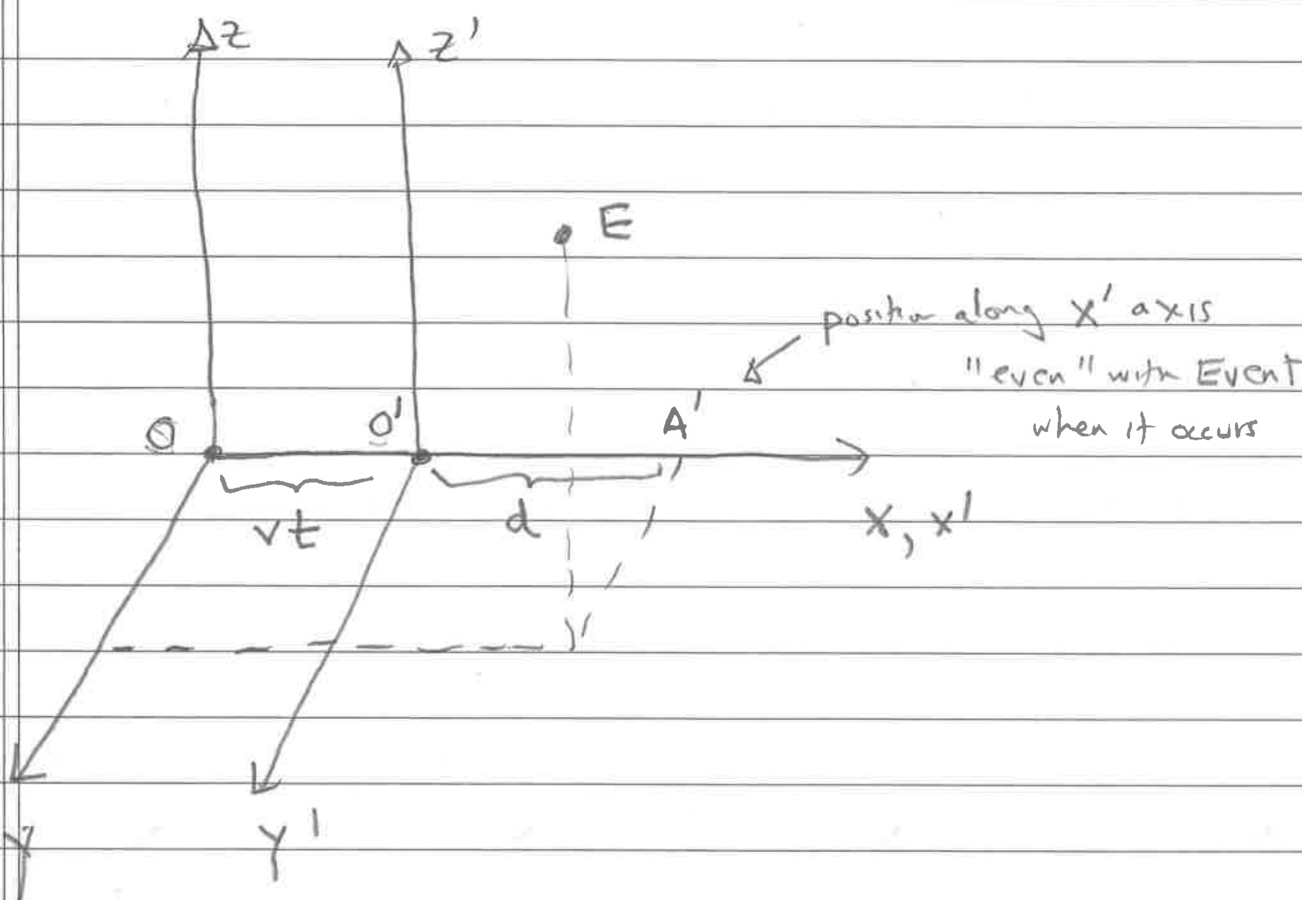
Which is higher?

R10

Lorentz Transformation (SKIP)

Event : something that takes place at specific location $(x\ y\ z)$ at specific time t

How are $(x'\ y'\ z')$ and t' in a frame S' moving wrt frame S related to $(x\ y\ z)$ and t



From viewpoint of person in S :

$x = d + vt$ and "obviously" $d = x'$

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$ ← of course time is same !

R11

But $d \neq x'$!

d is the distance between O' and A' in S

Whereas x' is distance between O' and A' in S'

O' and A' are at rest in S', so x' appears

contracted in S: $d = \frac{x'}{\gamma}$ $x' = d\gamma$

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} > 1$

$\frac{1}{\gamma} = \sqrt{1-\frac{v^2}{c^2}} < 1$

$x' = \gamma(x - vt)$

$y' = y$

$z' = z$

$t' = \gamma\left(t - \frac{vx}{c^2}\right)$

$x = \gamma(x' + vt')$

$y = y'$

$z = z'$

$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$

This follows from using the symmetrical reln

$x = \gamma(x' + vt')$

$= \gamma(\gamma(x - vt) + vt')$

$= \gamma^2 x - \gamma^2 v t + \gamma v t'$

$t' = \frac{\gamma^2 v t}{\gamma v} + \frac{1}{\gamma v}(1-\gamma^2)x$

$1-\gamma^2 = -\frac{v^2}{c^2}\gamma^2$

$t' = \gamma t + \gamma \frac{vx}{c^2}$

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

$\gamma^2 = \frac{1}{1-v^2/c^2}$

$1-\gamma^2 = 1 - \frac{1}{1-v^2/c^2}$

$= \frac{-v^2/c^2}{1-v^2/c^2} = -\frac{v^2}{c^2}\gamma^2$

R12

Can rediscover relativity of simultaneity, time dilation, and Lorentz contraction from Lorentz transformation

SIMULTANEITY

Event A $\quad x_A = 0 \quad t_A = 0$

Event B $\quad x_B = b \quad t_B = 0$

$$\rightarrow \begin{cases} x_A' = \gamma(x_A - v t_A) = 0 \\ t_A' = \gamma\left(t_A - \frac{v x_A}{c^2}\right) = 0 \end{cases}$$

$$\rightarrow \begin{cases} x_B' = \gamma(x_B - v t_B) = \gamma b \\ t_B' = \gamma\left(t_B - \frac{v x_B}{c^2}\right) = -\frac{\gamma v b}{c^2} \end{cases}$$

$\leftarrow$ B occurs <u>before</u> A

DILATION (skip)

If we watch a clock in S' at <u>fixed</u> position x' and compare time between two events

$$t_B = \gamma\left(t_B' + \frac{v x_B'}{c^2}\right)$$

$$t_A = \gamma\left(t_A' + \frac{v x_A'}{c^2}\right)$$

$x_A' = x_B'$

$$t_B - t_A = \gamma(t_B' - t_A')$$

$$\Delta t = \gamma \Delta t' \qquad \Delta t' = \frac{1}{\gamma}\Delta t$$

R13

Lorentz Contraction

Can also be verified

Einstein's Velocity addition Rule

R14

write Lorentz transformation in matrix form

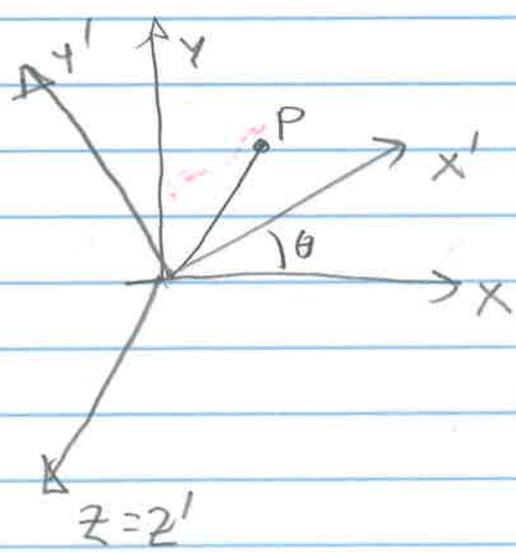
$$x^0 = ct$$
$$x^1 = x$$
$$x^2 = y$$
$$x^3 = z$$

$\Lambda^\mu{}_\nu$ Lorentz transformation matrix

$$\begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}' = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

of course reminiscent of rotation eg about z axis

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}' = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



$$\underbrace{(a^\mu)' = \sum_{\nu=0}^{3} \Lambda^\mu{}_\nu a^\nu}$$

transformation rule for any "4 vector"

R15

Similarity even more evident

$\cosh\theta = \gamma$

$\sinh\theta = \beta\gamma$

$$\left(\frac{e^{\theta}+e^{-\theta}}{2}\right)^2 + \left(\frac{e^{\theta}-e^{-\theta}}{2}\right)^2$$

$\cosh^2\theta - \sinh^2\theta = 1$

$\gamma^2 - \beta^2\gamma^2 = \gamma^2(1 - v^2/c^2) = 1$ ✓

Rotation: $(x^2+y^2+z^2)' = (x^2+y^2+z^2)$

Length of vector invariant

$$(-a^0b^0 + a^1b^1 + a^2b^2 + a^3b^3)'$$

"contravariant" $= (-a^0b^0 + a^1b^1 + a^2b^2 + a^3b^3)$

keep track of sign by introducing "covariant"

$a_\mu = (a_0\ a_1\ a_2\ a_3) = (-a^0\ a^1\ a^2\ a^3)$

$a_\mu = g_{\mu\nu}a^\nu$ $\qquad g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

"Minkowski Metric"

R16

$\sum_{\mu} a^{\mu} a_{\mu}$ is invariant under Lorentz transformation

$\therefore$ sensible to characterize

all observers will agree
$$\begin{cases} a^{\mu} a_{\mu} > 0 & \text{spacelike} \\ a^{\mu} a_{\mu} = 0 & \text{light like} \\ a^{\mu} a_{\mu} < 0 & \text{timelike} \end{cases}$$

$\Delta x^{\mu} = x^{\mu}_{A} - x^{\mu}_{B}$

timelike: there is an inertial system where 2 events occur at <u>same point</u>

spacelike: can find inertial system where events occur at same time (simultaneous)

R17

Relativistic momentum

$$\vec{p} = m\vec{v} / \sqrt{1 - v^2/c^2} = \gamma m \vec{v}$$

$$E = cp^0 = mc^2 / \sqrt{1 - v^2/c^2} = \gamma m c^2$$

p^μ is a 4 vector like x^μ obeying

same transformation eqns.

<u>Expt fact</u> : With these definitions energy and momentum are conserved. This rests on expt just as Lorentz transformation rests on Michelson Morley demonstration that c is same in all reference frames

$$p^\mu p_\mu = -(p^0)^2 + (p^1)^2 + (p^2)^2 + (p^3)^2$$

$$= -\gamma^2 m^2 c^2 + m^2 \frac{v^2}{1 - v^2/c^2}$$

$$= +\gamma^2 c^2 m^2 \underbrace{\left(-1 + v^2/c^2\right)}_{-1/\gamma^2} = -m^2 c^2$$

$$E^2 - c^2 p^2 = m^2 c^4$$

For a photon (massless particle) $E = cp$

R17A

Distinguish invariant ← same in all inertial frames

conserved ← same before and after some process in given frame

m : invariant, not conserved

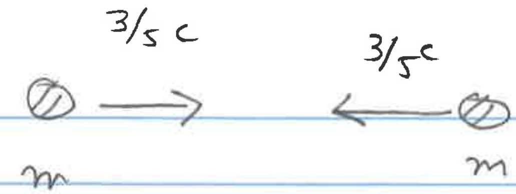
E : conserved, not invariant

q : conserved and invariant

V : not conserved, not invariant

R18

1.



Conservation of momentum is trivial : zero before + after

Energy before $\quad mc^2/\sqrt{1-(3/5)^2} = \frac{5}{4}mc^2$

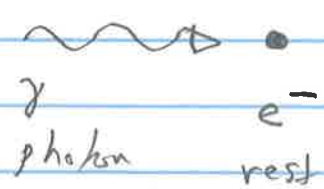
$\times 2$

Energy after $\quad Mc^2 = \frac{5}{2}mc^2$

$M = 5/2\, m \leftarrow$ more than $2m$!

"thermal energy" or "internal energy" represented as mass of composite object.

2. photon energy E_0 scattering off e^- at rest



Goal: relate change in λ to scattering angle θ

$\Updownarrow$

$p_e, p_\gamma, \theta, \phi$ + 3 eqns

y: $\quad p_e \sin\phi = p_\gamma \sin\theta$

$$\sin\phi = \frac{E}{p_e c}\sin\theta$$

R19

$$\overset{p_\gamma \text{ originally}}{}$$

X: $p_\gamma \cos\theta + p_e \cos\phi = E_0/c$

$$\frac{E}{c}\cos\theta + p_e\sqrt{1-\left(\frac{E}{p_e c}\sin\theta\right)^2} = E_0/c$$

$\cos^2\phi = 1-\sin^2\phi$

$$= 1-\left(\frac{E}{p_e c}\sin\theta\right)^2$$

$$p_e^2\left(1-\frac{E^2}{p_e^2c^2}\sin^2\theta\right) = \frac{(E_0 - E\cos\theta)^2}{c^2}$$

$$p_e^2c^2 = (E_0 - E\cos\theta)^2 + E^2\sin^2\theta$$

$$= E_0^2 - 2EE_0\cos\theta + E^2$$

Energy conservation

$$E_0 + m_ec^2 = E + \sqrt{m_e^2c^4 + p_e^2c^2}$$

$$= E + \sqrt{m_e^2c^4 + E_0^2 - 2EE_0\cos\theta + E^2}$$

solve for $E = \dfrac{1}{(1-\cos\theta)/mc^2 + 1/E_0}$

Using Definition $E = h\nu = hc/\lambda$

$$\lambda = \lambda_0 + \frac{h}{mc}(1-\cos\theta)$$

$\boxed{\text{Compton Scattering}}$

R19A

$$(E_0 + m_e c^2 - E) = \sqrt{m_e^2 c^4 + E_0^2 - 2EE_0\cos\theta + E^2}$$

$$\left.\begin{array}{l} E_0^2 + m_e^2 c^4 + E^2 \\ +2E_0 m_e c^2 - 2E_0 E \\ -2E m_e c^2 \end{array}\right\} = \begin{array}{l} m_e^2 c^4 + E_0^2 + E^2 \\ -2EE_0\cos\theta \end{array}$$

$$\Rightarrow \quad E_0 m_e c^2 = E\left(E_0 + m_e c^2 - 2E_0\cos\theta\right)$$

$$E = \frac{E_0 m_e c^2}{E_0(1-\cos\theta) + m_e c^2}$$

$$= \frac{1}{\frac{1-\cos\theta}{m_e c^2} + \frac{1}{E_0}}$$