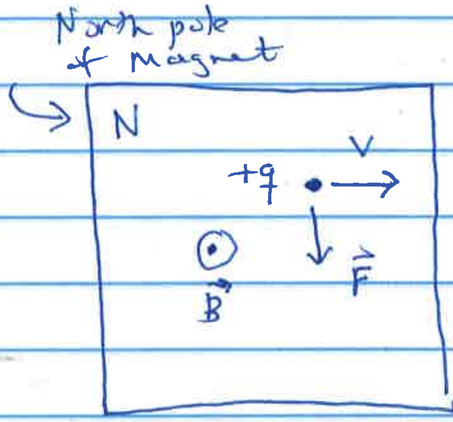
F1

We have considered magneto- and electro- statics

What if $\partial\vec{B}/\partial t$ and/or $\partial\vec{E}/\partial t$ nonzero?

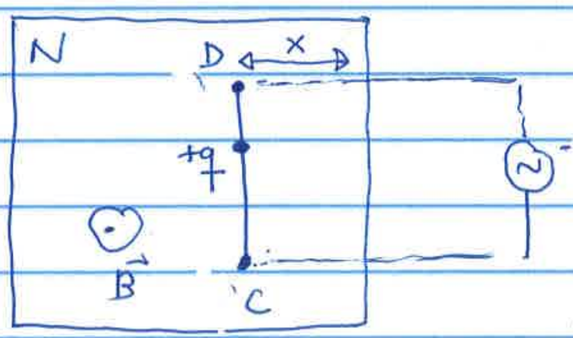



$$\vec{F} = q\vec{v}\times\vec{B}$$

Suppose, however we considered the equivalent problem of moving the magnet with +q at rest

Since charges at rest experience only electric forces, we are forced to conclude the moving magnet ( $\partial\vec{B}/\partial t \neq 0$ ) produces an electric field

We can think about the charge +q as being in a wire and then measuring the potential between the ends of the wire by attaching a voltmeter (more wires)




$\oint E\cdot dl$

F2

We know the work done as calculated from the original point of view of pushing against the magnetic force $W = qvB\ell$

So the potential is $\frac{W}{q} = \oint \vec{E}\cdot\vec{d\ell} = vB\ell$

Switching to the viewpoint of the magnet moving

we notice that

$\Phi_B = B\ell x$

$d\Phi_B/dt = B\ell \frac{dx}{dt}$

$= B\ell v$

$\oint E \cdot \vec{d\ell} = d\Phi_B/dt$ at least in magnitude

In fact $d\Phi_B/dt < 0$ so

$$\oint \vec{E}\cdot\vec{d\ell} = -d\Phi_B/dt$$

Although derived for a particular geometry, this law is true in general.

$E(4\pi r^2)$

$\ast$ $q$ $E = \frac{1}{4\pi\epsilon_0} q/r^2 \Rightarrow \Phi_E = q/\epsilon_0$

specific geometry general law

F3

Rewrite using Gauss' Law

$$-\frac{\partial \phi_B}{\partial t} = \oint \vec{E} \cdot \vec{dl}$$

$$-\frac{\partial}{\partial t} \int_S \vec{B} \cdot \hat{n}\, dA = \int_S (\vec{\nabla} \times \vec{E}) \cdot \hat{n}\, dA$$

$$-\partial B/\partial t = \vec{\nabla} \times \vec{E}$$

Previously $\vec{\nabla} \times \vec{E} = 0$

$$\vec{E} = -\vec{\nabla}\phi$$

Since

$$\boxed{\begin{aligned} \vec{B} &= \vec{\nabla} \times \vec{A} \\ \vec{E} &= -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t} \end{aligned}}$$

Recall one more thing ... $\vec{\nabla} \times \vec{B} = \mu_0 J$ ,

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu_0 \vec{J}$$

$$-\nabla^2 \vec{A} + \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) = \mu_0 \vec{J}$$

Choose ("Coulomb gauge") $\vec{\nabla} \cdot \vec{A} = 0$

$$-\nabla^2 \vec{A} = \mu_0 \vec{J}$$

Agreed with our magnetostatic starting point that dipole $\Rightarrow \vec{B}$ field

$$\Rightarrow \vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(r')}{|\vec{r} - \vec{r}'|} d^3 r'$$