EM I

Electrostatic fields in the presence of matter

We have discussed electric fields of point charges q_i and of charge distributions $\rho(\vec{r})$, perhaps subject to certain bdy conditions like the presence of a metal which forces $\vec{E}$ to be $\perp$ to surface.

We now turn more generally to the effect of the presence of matter on $\vec{E}$ fields (and, later, $\vec{B}$ fields). In a sense this is not a new topic at all, because matter is just a collection of charges.

However, it is often convenient to isolate the "free charges" eg over which we have experimental control, from the charges in a material which are responding to the free charges.

Indeed, if you think about it, treating the true $\vec{E}, \vec{B}$ fields in matter is going to be impossible – vast numbers (10^{23}) of q_i, $\vec{E}$ which wildly vary in space and in time. Inevitably we need to average these microscopic fluctuations (unless our expt probes them explicitly, as some do!). This is our motivation for developing models of how matter affects $\vec{E}$ fields.

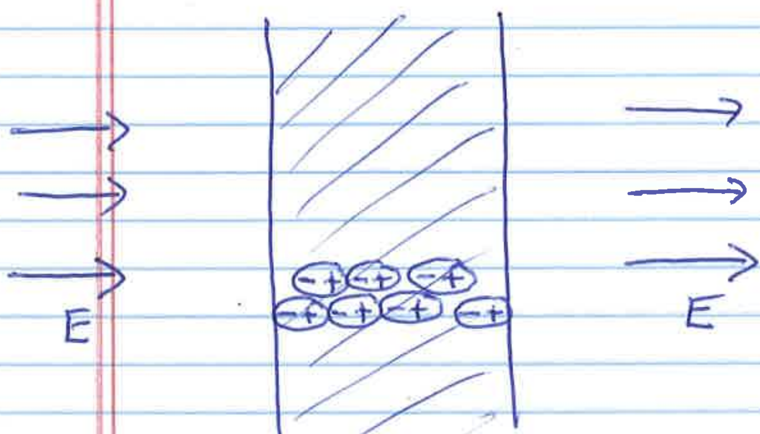
EM2

The simplest model is to treat matter as a "dielectric";

we imagine material

- has no free charge; all charges attached to specific atoms/molecules
- $\vec{E}$ fields produce small displacements from equilibrium
- Effect of $\vec{E}$ can be viewed as displacement of all + charge of dielectric relative to − charge $\Rightarrow$ material is "polarized"

Physical picture:



accumulation of − charge

accumulation of + charge

looks as if ^+Q is reduced as in case at left

$\boxed{Q:}$ what about outside

always (Le Chatelier) $\Rightarrow$ $\leftarrow$ Induced E inside opposes external E

Our goal: compute total $\vec{E}$ inside

EM2'

Is it reasonable to think displacements are small?
Can we rip e^- away from p.?

$\vec{E}$ field in an atom

$$\vec{E} = \frac{1}{4\pi\epsilon_0} q/r^2$$

$$= 9 \cdot 10^9 \, 1.6 \cdot 10^{-19} / (.5 \cdot 10^{-10})^2 \sim 10^{12} \frac{V}{m} \cdot \frac{N}{c}$$

$\vec{E}$ field applied? Anyone with lab experience?

"Household experience" 120 V wire $\sim$ 1 m long $\sim 10^2 \frac{V}{m}$

Can thermal effects rip e^- from p?

$$\text{Energy} \sim 13.6 \text{ eV} \rightarrow 10^5 \text{ °K}$$

$$1 \text{ eV} = 12000 \text{ °K}$$

But some materials (eg semiconductors) binding energy is less and can see effect of T on # of carriers.

Electric field in laser

Energy density $\frac{1}{2}\epsilon_0 E^2$

Multiply by speed of light $\quad \frac{1}{2}\epsilon_0 E^2 c = \frac{\text{Energy}}{\text{area} \cdot \text{time}}$

$$\frac{1}{4\pi\epsilon_0} = 9 \cdot 10^9 \qquad \epsilon_0 = 8.85 \cdot 10^{-12}$$

EM2''

National Ignition Facility $\qquad 10^{19}\ \frac{\text{Watts}}{m^2}$

NIF

Solar power $\qquad 1\ \frac{\text{KW}}{m^2}$

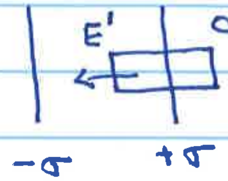
$$10^3 = \frac{1}{2}(8.85\cdot 10^{-12})\, E^2\, 3\cdot 10^8$$

$$E \sim 10^3\ V/m$$

Cell phone? Is it dangerous?

EM 3

We could get $\bar{E}$ inside if we knew the size of the accumulated charge at the surface



$$E_{inside} = E - \sigma/\epsilon_0$$

Our basic assumption is $\vec{p} = \gamma \vec{E}$ ← applied field

p: induced dipole moment. γ: polarizability

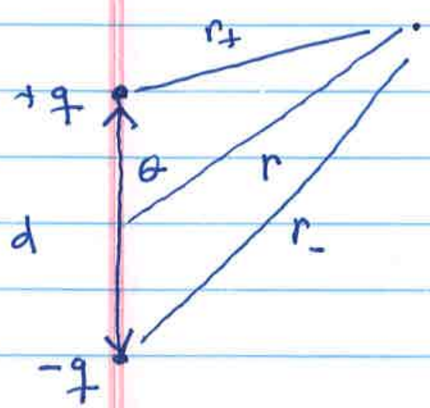
We will write down some simple models which allow us to compute γ later, but this is a reasonable assumption

basically just $F = kd$ ← some sort of restoring force on displaced charge

$$Eq = kd = k\,P/q \quad \text{so} \quad p \sim E$$

EM 4

It is useful to review $\vec{E}$ field and potential V due to a dipole



$$V(r) = \frac{1}{4\pi\epsilon_0}\left\{\frac{q}{r_+} - \frac{q}{r_-}\right\}$$

$$r_+^2 = r^2 + d^2 - 2rd\cos\theta$$

$$r_-^2 = r^2 + d^2 + 2rd\cos\theta$$

Expanding
$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{qd\cos\theta}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3} \qquad |\vec{p}| = qd$$

also
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{3\hat{r}(\vec{p}\cdot\hat{r}) - \vec{p}}{r^3}$$

For a distribution of dipoles

$$V(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\vec{p}(\vec{r}\,')\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\,d^3r'$$

$$= \frac{1}{4\pi\epsilon_0}\int \vec{p}(\vec{r}\,')\cdot\nabla'\frac{1}{|\vec{r}-\vec{r}\,'|}\,d^3r'$$

EM5

integrating by parts

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left\{ \int \nabla' \cdot \frac{\vec{P}(r')}{|\vec{r}-\vec{r}'|}\, d^3r' - \int \frac{\vec{\nabla}' \cdot \vec{P}(\vec{r}')}{|\vec{r}-\vec{r}'|}\, d^3r' \right\}$$

divergence theorem

$$= \frac{1}{4\pi\epsilon_0}\left\{ \int \frac{\vec{P}(\vec{r}')\cdot \hat{n}'}{|\vec{r}-\vec{r}'|}\, da' - \int \frac{\nabla' \cdot P(r')}{|\vec{r}-\vec{r}'|}\, d^3r' \right\}$$

$\sigma_b \equiv \vec{P}(r') \cdot \hat{n}'$ the surface charge density

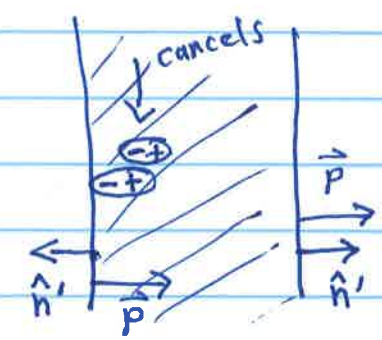
$\rho_b \equiv -\vec{\nabla}' \cdot \vec{P}(r')$ the bound charge density

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left\{ \int \frac{\sigma_b(r')}{|\vec{r}-\vec{r}'|}\, da' + \int \frac{\rho_b(r')}{|\vec{r}-\vec{r}'|}\, d^3r' \right\}$$

Turning to our simple geometry, since $\vec{\nabla}\cdot\vec{E} = 0$,

and $\vec{P} = \gamma\vec{E}$ we have $\vec{\nabla}\cdot\vec{P} = 0$, no bound charge density

Physically:



Meanwhile $\sigma_b = \vec{P}\cdot\hat{n}'$

$= -\gamma E$ on left

$+\gamma E$ on right

to finish problem someone needs to tell us material property, namely a value for γ

EM 6

$$\rho_{tot} = \underset{\text{free}}{\rho_f} + \underset{\text{bound}}{\rho_b}$$

$$\epsilon_0 \vec{\nabla}\cdot\vec{E} = \rho_f - \vec{\nabla}\cdot\vec{P}$$

$$\vec{\nabla}\cdot(\epsilon_0\vec{E} + \vec{P}) = \rho_f$$

$\vec{D}$, the electric displacement

one usually writes $\vec{P} = \epsilon_0 \overbrace{\chi_e}^{\gamma \text{ earlier}} \vec{E}$ electric susceptibility

$$\vec{D} = \epsilon_0 \underbrace{(1+\chi_e)}_{\text{"dielectric constant" } \epsilon/\epsilon_0} \vec{E} = \epsilon\vec{E}$$

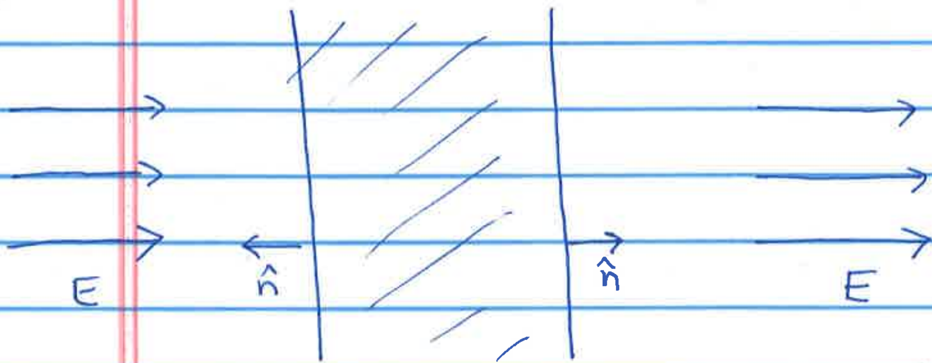
we will see later

$\vec{M} = \chi_m \vec{B}$ similarly

magnetic susceptibility

$\vec{P} = \epsilon_0 \chi_e \vec{E}$

electric susceptibility



$\sigma_b = \hat{P}\cdot\hat{n}$ $\quad \epsilon \qquad \sigma_b = \vec{P}\cdot\hat{n} = +\epsilon_0 \chi_e E$

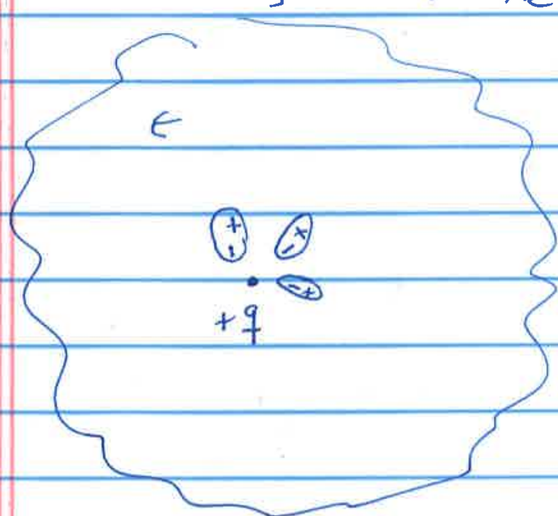
$= -\epsilon_0 \chi_e E$

$\rho_b = -\vec{\nabla}\cdot\vec{P} = -\vec{\nabla}\cdot \underset{\text{if constant}}{\epsilon_0 \chi_e} \vec{E} \quad -\epsilon_0 \chi_e \underbrace{\vec{\nabla}\cdot\vec{E}}_{\rho/\epsilon_0} = -\chi_e \rho$

$\rho_b = -\chi_e (\rho_b + \rho_f)$

$\rho_b = -\chi_e \rho_f / 1+\chi_e$ $\qquad \boxed{\text{if } \rho_f = 0 \text{ so too is } \rho_b\,!}$

$\rho = \rho_f / 1+\chi_e$



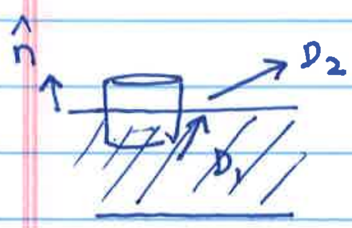
Polarization cloud screens +q

$\rho = \rho_f / 1+\chi_e$

total charge density is reduced from ρ_f

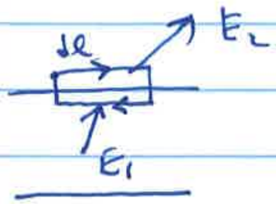
EM7

Boundary conditions since $\vec{\nabla}\cdot\vec{D} = \rho_f$ or free



The discontinuity of D_n is proportional to free surface charge density

$$D_{2n} A - D_{1n} A = \sigma_{free} A$$



$$\vec{E}_2\cdot\vec{dl} + \vec{E}_1\cdot(-\vec{dl}) = 0$$

$$E_{2t} = E_{1t}$$

EM8

$$\vec{\nabla} \times \vec{E} = 0 \text{ still holds}$$

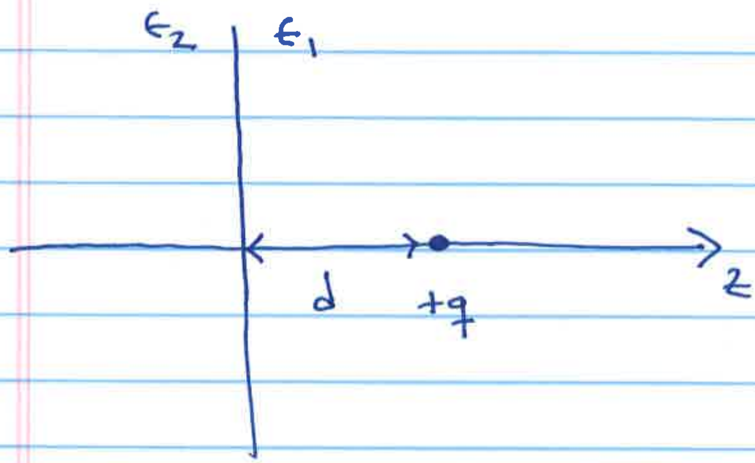
$$\vec{\nabla} \cdot \vec{E} = \rho_f/\epsilon \qquad \vec{E} = -\vec{\nabla} V$$

$$\nabla^2 V = -\rho_f/\epsilon$$

if no free charge density still have laplaces eqn

$$\nabla^2 V = 0$$

So all our old techniques still work, we just need to make sure we adapt them to correct bdy conditions!



point charge near interface of two different dielectric media (often one of ϵ_1, ϵ_2 is ϵ_0, ie vacuum)

$$\left.E_x\right|_{z=0^+} = \left.E_x\right|_{z=0^-}$$

$$\left.E_y\right|_{z=0^+} = \left.E_y\right|_{z=0^-}$$

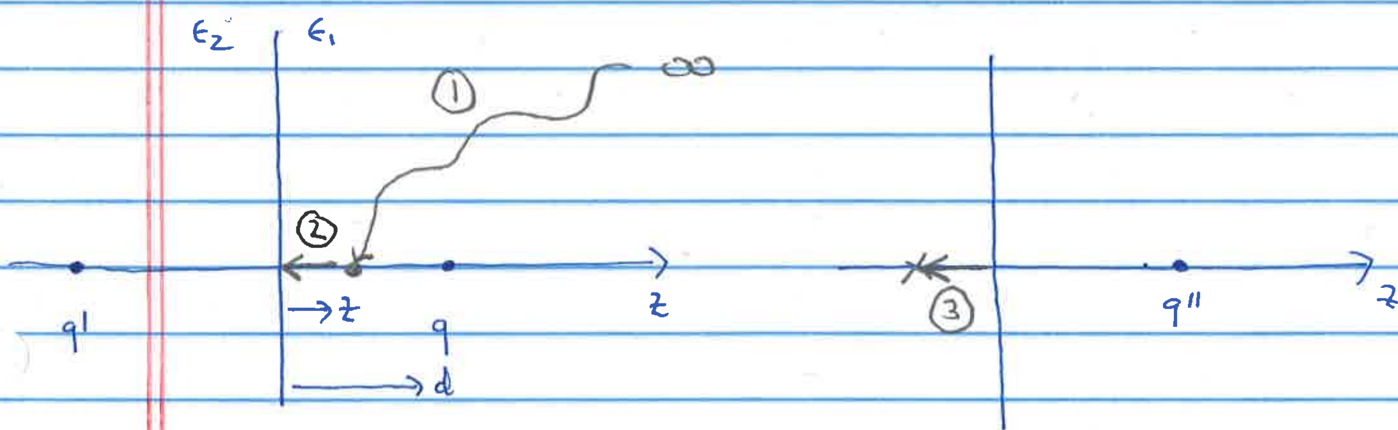
} tangential components of E

$$\epsilon_1 \left.E_z\right|_{z=0^+} = \epsilon_2 \left.E_z\right|_{z=0^-}$$

} normal components of D

Michael asks (I think):

"what about expectation that V will depend on both ϵ_1 and ϵ_2 because V represents work done bringing test charge in from ∞ and the path lies partly in region ① and partly in region ②?"



$z > 0$ $z < 0$

Let's do the calculation

we do not need to do this integral! we are guaranteed it ~~we are~~ is just ΔV and indep. of path!

$$\underset{\substack{\uparrow \\ 0}}{V_1 - V_\infty} = -\int_\infty^1 \vec{E}_1 \cdot d\vec{l} = q/4\pi\epsilon_0 \underset{\substack{\uparrow \\ d-z}}{R_1} + q'/4\pi\epsilon_1 \underset{\substack{\uparrow \\ d+z}}{R_2}$$

$$V_2 - V_\infty = V_2 - V_1 + V_1 - \infty = q/4\pi\epsilon_0 \underset{\substack{\uparrow \\ d}}{R_1} + q'/4\pi\epsilon_1 \underset{\substack{\uparrow \\ d}}{R_2}$$

QM 1

$\boxed{\text{Connection to QM}}$

Hydrogen atom $\qquad a_0 = 4\pi\epsilon_0 \hbar^2/m_e e^2$

1 0 0 $\qquad 1/\sqrt{\pi}\;\; 1/a_0^{3/2}\;\; e^{-r/a_0}$

2 0 0 $\qquad 1/4\sqrt{2\pi}\;\; 1/a_0^{3/2}\; (2 - r/a_0)\; e^{-r/a_0}$

2 1 0 $\qquad 1/4\sqrt{2\pi}\;\; 1/a_0^{3/2}\;\; r/a_0\;\; e^{-r/2a_0}\;\; \cos\theta$

2 1 ±1 $\qquad 1/8\;\; 1/\sqrt{\pi}\;\; 1/a_0^{3/2}\;\; r/a_0\;\; e^{r/2a_0}\;\; \sin\theta\; e^{\pm i\phi}$

$R_{\ell n}(r)\, Y_{\ell m}(\theta,\phi)$ familiar

$E_{n\ell m} = e^2/2a_0\;\; 1/n^2$

indep of ℓ, m

200, 210, 21±1 all equally likely

→ spherically symmetric

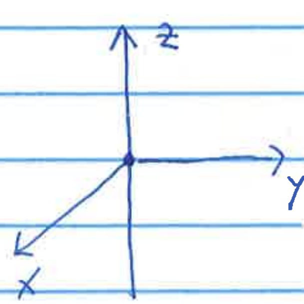
apply $\vec{E} = E_0 \hat{z}$ $\qquad$ angular structure

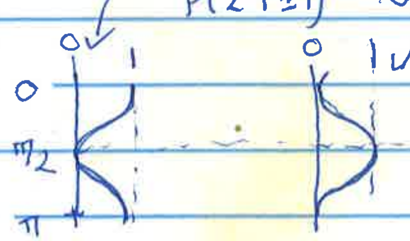
$P(2,0,0) \sim 1$

$P(210) \sim \cos^2\theta$

$P(21\pm1) \sim \sin^2\theta$

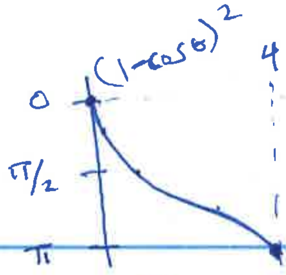
$\theta = 0, \pi$ Equally likely...

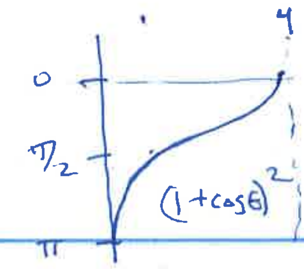




$\boxed{Q}$

QM 2





$\boxed{A}$ Linear combination!

$|200\rangle - |210\rangle \sim 1-\cos\theta$ ← peaked at $\theta = \pi$

$|200\rangle + |210\rangle \sim 1+\cos\theta$ ← peaked at $\theta = 0$

$\boxed{Q}$ which is lower for electron? ← $\boxed{A}$ negatively charged should move to $z<0$ (opposite to E) hence $\theta = \pi$

$\boxed{Q}$ Did you do this problem before? How?

$\boxed{A}$ Yes. Stark effect eg by perturbation theory

$H^{(1)} = -eE_0 z$ $\qquad E^{(1)}_{n\ell m} = -eE_0 \langle n\ell m | z | n\ell m\rangle = 0$

Eg 100 $\quad \overbrace{\int \sin\theta\, d\theta}^{d^3r}\, \underset{\downarrow}{\cos\theta}$ (z)

Conclude no first order shift

210 $\quad \int \sin\theta\, d\theta \cos\theta \cos^2\theta$

$\boxed{Q}$ is that right?!?

No! If states degenerate need to diagonalize matrix

$$\begin{array}{c|c} & \begin{matrix} 200 & 210 & 211 & 21{-1} \end{matrix} \\ \begin{matrix} 200 \\ 210 \\ 211 \\ 21{-1} \end{matrix} & \begin{pmatrix} 0 & x & 0 & 0 \\ x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

QM3

perturbed ground state is linear combination of $|200\rangle$ $|210\rangle$

as we intuitively expected..

Details: $\langle 200|z|210\rangle$ angular part

$$\sim \underbrace{\int d\theta \sin\theta}_{d^3r} \; \underset{200}{1} \; \underset{z}{\cos\theta} \; \underset{210}{\cos\theta} \sim -\frac{1}{3}\cos^3\theta\Big|_0^\pi \neq 0$$

$$-\tfrac{1}{3}(-1-1)$$

$\langle 200|z|211\rangle$

$$\sim \underbrace{\int d\theta \sin\theta}_{d^3r} \; \underset{200}{1} \; \underset{z}{\cos\theta} \; \underset{211}{\sin\theta} \; \underbrace{\int d\phi}_{\text{vanishes also!}} e^{i\phi}$$

$$\sim \frac{\sin^3\theta}{3}\Big|_0^\pi = 0$$

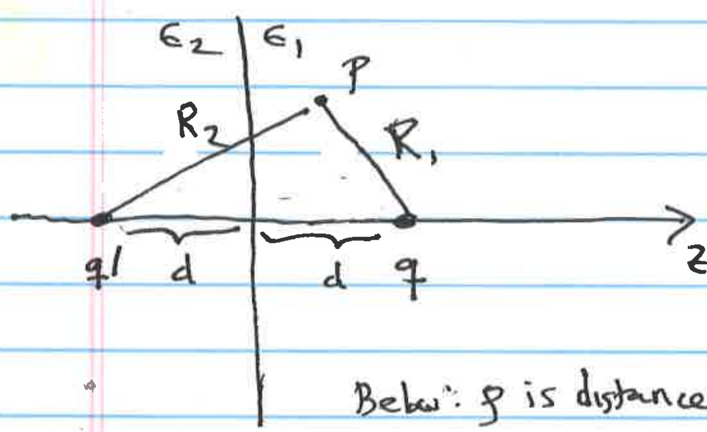
etc.

EM 9

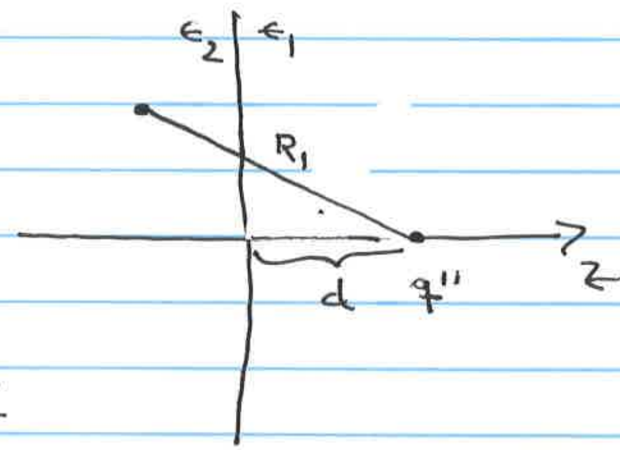
$\boxed{Q}$ Any ideas for method?

$\boxed{A}$ Images ← Rules of game: can monkey around with $\rho(\vec{r})$ outside region of V to heart's delight.

a) $z > 0$



b) $z < 0$



Below: ρ is distance $\sqrt{x^2+y^2}$

$\boxed{Q}$ why are we forced to use "real" q for $z>0$ but allowed fictitious q'' for $z<0$

$$V = q/4\pi\epsilon_1 R_1 + q'/4\pi\epsilon_1 R_2 \qquad V = q''/4\pi\epsilon_2 R_1$$

tangential E

$$-\frac{1}{4\pi\epsilon_1}\frac{\partial}{\partial\rho}\left(q/R_1 + q'/R_2\right)\Big|_{z=0} = -\frac{1}{4\pi\epsilon_2}\frac{\partial}{\partial\rho}\, q''/R_1\Big|_{z=0}$$

$$1/\epsilon_1 \frac{q\rho + q'\rho}{(\rho^2+d^2)^{3/2}} = \frac{1}{\epsilon_2}\frac{q''\rho}{(\rho^2+d^2)^{3/2}}$$

$$1/\epsilon_1 (q+q') = \frac{1}{\epsilon_2} q''$$

NB This also makes the $z>0$ and $z<0$ forms for V equal at $R_1 = R_2$ ie along the dividing line where 2 regions touch.

EM10

Normal Component of D

$$-\frac{1}{4\pi}\frac{\partial}{\partial z}\left(\frac{q}{R_1}+\frac{q'}{R_2}\right)\Bigg|_{z=0} = -\frac{1}{4\pi}\frac{\partial}{\partial z}\frac{q''}{R_1}\Bigg|_{z=0}$$

$$\frac{qd - q'd}{(\rho^2+d^2)^{3/2}} = \frac{q''d}{(\rho^2+d^2)^{3/2}}$$

$$q - q' = q''$$

$$\hookrightarrow \quad q' = -\left(\frac{\epsilon_2-\epsilon_1}{\epsilon_2+\epsilon_1}\right)q$$

$$q'' = \left(\frac{2\epsilon_2}{\epsilon_2+\epsilon_1}\right)q$$

Surface charge density $\sigma_b = \vec{P}\cdot\hat{n}$

$\vec{D} = \epsilon\vec{E}$
$= \epsilon_0\vec{E}+\vec{P}$
$\vec{P} = (\epsilon-\epsilon_0)\vec{E}$

$$\sigma_b = \vec{P}_1\cdot(-\hat{e}_z) + \vec{P}_2\cdot\hat{e}_z = -P_{1z} + P_{2z}$$

$$P_z = (\epsilon-\epsilon_0)E_z$$

$$\sigma_b = (\epsilon_1-\epsilon_0)\frac{1}{4\pi\epsilon_1}\frac{d(q-q')}{(\rho^2+d^2)^{3/2}} - (\epsilon_2-\epsilon_0)\frac{1}{4\pi\epsilon_2}\frac{q''d}{(\rho^2+d^2)^{3/2}}$$

$$= -\frac{q}{2\pi}\frac{\epsilon_0(\epsilon_2-\epsilon_1)}{\epsilon_1(\epsilon_2+\epsilon_1)}\frac{d}{(\rho^2+d^2)^{3/2}}$$

Vanishes when $\epsilon_1 = \epsilon_2$ as expected!

Q: But shouldn't it be symmetric under $\epsilon_1 \to \epsilon_2$ somehow?!

Why ϵ_1 special?! $\boxed{A}$ Q is in region ①.

EM 11

$\boxed{Q}$ Any limit you can check

$\boxed{A}$ Metal $\quad \epsilon_2 \to$ large

$$q' = -\left(\frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + \epsilon_1}\right) q \to -q \quad \checkmark\checkmark$$

tangential E

$$\underbrace{\tfrac{1}{\epsilon_1}(q+q') = \tfrac{1}{\epsilon_2} q''}_{\text{both vanish}}$$

But

$$q'' = \left(\frac{2\epsilon_2}{\epsilon_2 + \epsilon_1}\right) q \longrightarrow 2q \quad ?? \ \boxed{Q}$$

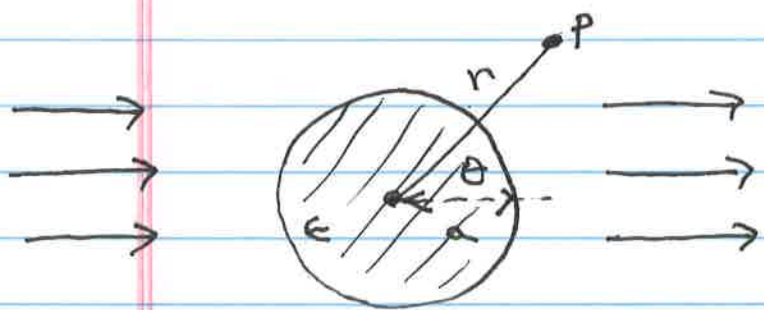
$\boxed{A}$ No problemo $\quad V$ for $z<0$ is $q''/4\pi\epsilon_2 R_1$

so value of q'' irrelevant $V \to 0$

since $\epsilon_2 \to \infty$

EM12

A classic problem: dielectric sphere in uniform $\vec{E}$ field



$r > a$ $\quad V_>(r) = \sum \left[B_\ell r^\ell + C_\ell r^{-(\ell+1)} \right] P_\ell(\cos\theta)$

$\downarrow$

$\emptyset$ except $-E_0 z$

$= -E_0 r \cos\theta$

$\uparrow$

$P_1(\cos\theta)$

$r < a$ $\quad V_<(r) = \sum A_\ell r^\ell P_\ell(\cos\theta)$

tangential E $\quad -\frac{1}{a} \frac{\partial V_>}{\partial \theta}\Big|_{r=a} = -\frac{1}{a} \frac{\partial V_<}{\partial \theta}\Big|_{r=a}$

usual argument: coefficients of $P_\ell(\cos\theta)$ on either side must match term-by-term.
The same is true even if $\partial/\partial\theta \, P_\ell(\cos\theta)$

$\ell = 1 \quad A_1 = -E_0 + C_1/a^3$

$\ell \neq 1 \quad A_\ell = C_\ell / a^{2\ell+1}$

EM 13

Normal D

$$-\epsilon \frac{\partial V_<}{\partial r}\Big|_{r=a} = -\epsilon_0 \frac{\partial V_>}{\partial r}\Big|_{r=a}$$

$$-\epsilon \sum_\ell A_\ell \, \ell \, a^{\ell-1} P_\ell(\cos\theta)$$

$$= \epsilon_0 E_0 \cos\theta + \epsilon_0 \sum_\ell (\ell+1) C_\ell \, a^{-(\ell+2)} P_\ell(\cos\theta)$$

$\ell = 1$ $\quad -\epsilon/\epsilon_0 \, A_1 = +E_0 + 2C_1/a^3$

$\ell \neq 1$ $\quad -\epsilon/\epsilon_0 \, A_\ell = (\ell+1) C_\ell / a^{2\ell+1}$

Obviousley $\quad A_\ell = C_\ell = 0 \quad \ell \neq 1$

and $\quad -\epsilon/\epsilon_0 \{-E_0 + C_1/a^3\} = +E_0 + 2C_1/a^3$

$$(-1 + \epsilon/\epsilon_0) E_0 = C_1/a^3 \, (2 + \epsilon/\epsilon_0)$$

$$C_1 = \left(\frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2}\right) E_0 a^3$$

$$A_1 = C_1/a^3 - E_0 = \frac{-3}{(\epsilon/\epsilon_0 + 2)} E_0$$

Finally

$\vec{E}, \vec{P}, \vec{D}$ constant inside sphere! $\sim \hat{z}$

$$\begin{cases} r<a & V_< = \dfrac{-3}{(\epsilon/\epsilon_0 + 2)} E_0 r \cos\theta \\ r>a & V_> = -E_0 r\cos\theta + \dfrac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2} E_0 \dfrac{a^3}{r^2} \cos\theta \end{cases}$$

check $\epsilon = \epsilon_0$ returns uniform E!

Notice at $r=a$ $\quad V_> = E_0 a \cos\theta \left\{ -1 + \frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2} \right\}$

EM14

$\frac{-\epsilon/\epsilon_0 - 2 + \epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2} = \frac{-3}{\frac{\epsilon}{\epsilon_0}+2}$

$\boxed{Q}$ Check limits ?

So $V_> = V_<$ @ $r=a$

$\boxed{A}$ $\epsilon = \epsilon_0 \Rightarrow$ uniform E_0 everywhere.

Dielectric sphere produces dipole moment

$$V = \frac{1}{4\pi\epsilon_0} \frac{\vec{p}\cdot\vec{r}}{r^3}$$

$$\Rightarrow \quad \vec{p} = 4\pi\epsilon_0 \frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2} E_0 a^3 \hat{z}$$

Alternately can get $\vec{P} = (\epsilon - \epsilon_0)\vec{E}$ inside sphere

$\vec{D} - \epsilon_0\vec{E}$

$$= 3/(\epsilon/\epsilon_0 + 2)\, E_0 \hat{z}$$

and then $\vec{p} = \frac{4}{3}\pi a^3 \vec{P} = 4\pi\epsilon_0 \frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2} E_0 a^3$ $\Downarrow$

same

Uniform $E_0 \Rightarrow$ constant $\underline{P}$ inside sphere $\Rightarrow \rho_b = 0$

$\nabla \cdot \underline{P}$

surface charge density $\quad \sigma_b = \underline{P}\cdot\hat{n} = \underline{P}\cdot\frac{\vec{r}}{r}$

$$= 3\epsilon_0 \frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2} E_0 \cos\theta$$

EM 15

$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0 = (\rho_f + \rho_b)/\epsilon_0$

lines of $\vec{E}$ terminate on ρ_b and ρ_f

$\vec{\nabla}\cdot\vec{P} = -\rho_b$

lines of $\vec{P}$ terminate on ρ_b

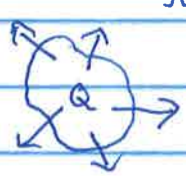
$\vec{\nabla}\cdot\vec{D} = \rho_f$

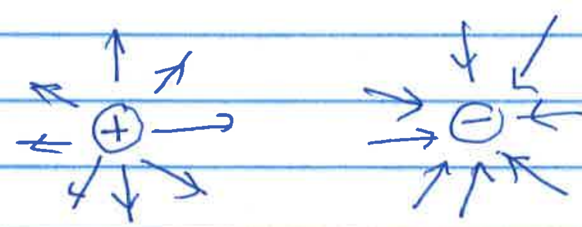
lines of $\vec{D}$ terminate on ρ_f

This is pictorial way of envisioning divergence theorem!

$$\Phi_E = \int \vec{E}\cdot\hat{n}\,dA = \int \vec{\nabla}\cdot\vec{E}\,dV = \frac{1}{\epsilon_0}\int \rho\,dV = Q/\epsilon_0$$

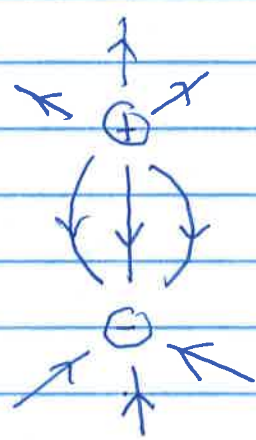
flux of $\vec{E}$ through surface





Monopoles

Dipole



EM16

For our dielectric sphere problem

$r<a$ $\quad V_< = \dfrac{-3}{(\epsilon/\epsilon_0+2)} \overbrace{E_0 r\cos\theta}^{z}$

$r>a$ $\quad V_> = -\underbrace{E_0 r\cos\theta}_{z} + \dfrac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0+2} E_0 \dfrac{a^3}{r^2}\cos\theta$

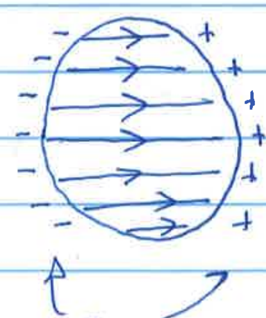
Inside sphere $\quad \vec{E} = -\vec{\nabla} V = \dfrac{-3}{(\epsilon/\epsilon_0+2)} E_0 \hat{z}$

and $\quad \vec{P} = (\epsilon-\epsilon_0)\vec{E}$ $\quad$ all are constant

$\vec{D} = \epsilon\vec{E}$ $\quad$ and in $\hat{z}$ direction

Outside sphere $\quad \vec{P} = (\epsilon_0-\epsilon_0)\vec{E} = 0$

So can draw $\vec{P}$



σ_b

+ dipole field outside

lines start and end on σ_b

$\boxed{Q}$ Why do they go from $\ominus$ to $\oplus$!?

$\boxed{A}$ $\vec{\nabla}\cdot\vec{P} = -\rho_b$

EM17

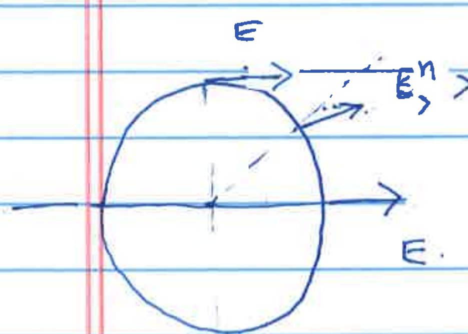
lines of $\vec{E}$ $\qquad V_< = \frac{-3}{(\epsilon/\epsilon_0+2)} E_0 r\cos\theta$

$$E_>^t = E_<^t = -\frac{1}{r}\,\partial V_</\partial\theta = -3/(\epsilon/\epsilon_0+2)\, E_0 \sin\theta$$

$$E_<^n = -\frac{\partial V_<}{\partial r} = 3/(\epsilon/\epsilon_0+2)\, E_0\cos\theta$$

D^n is continuous so $\epsilon E_<^n = \epsilon_0 E_>^n$

$$\Rightarrow E_>^n = \frac{\epsilon}{\epsilon_0} E_<^n$$

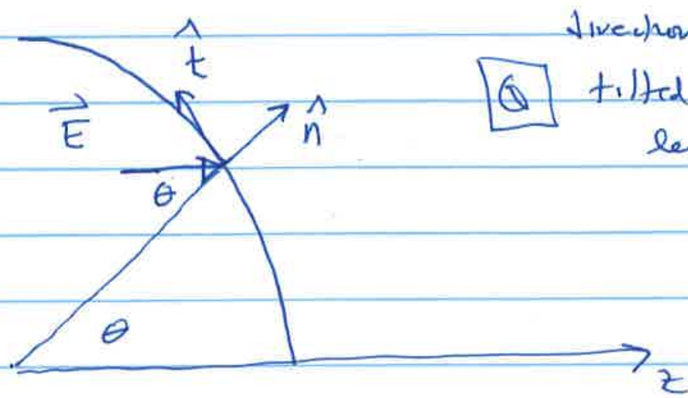


E has kink at dielectric surface since its normal component is reduced abruptly on entering the dielectric

	θ: 0	$\pi/4$	$\pi/2$
$E_>^t = -3/(\epsilon/\epsilon_0+2)\, E_0 \sin\theta$	0		
$E_>^n = 3\,\epsilon/\epsilon_0 \,/(\epsilon/\epsilon_0+2)\, E_0 \cos\theta$			0

normal component lengthened a little so not in $\hat{z}$ direction any more

$\boxed{\theta}$ tilted towards $\hat{n}$ will lengthen E_n



CM I

Polarization of single atom

Stark Effect perturbation $H^{(1)} = -eEz$

$\Rightarrow$ shifted wave functions with $\langle z \rangle \neq 0$

$\Rightarrow$ Polarization $\vec{p} = -e\underbrace{\langle z \rangle}_{\text{some number times } E}\hat{z}$

$\boxed{Q:}$ Is this what we add to $\epsilon_0\vec{E}$ to get $\vec{D}$

$\boxed{A}$ Dimensionally wrong

$$[\epsilon_0 E] = Q/L^2 \qquad [\vec{p}] = QL$$

Missing $1/L^3$. What is it?

Physically the density of atoms must be relevant

$$\vec{P} = -en\langle z \rangle \hat{z}$$

factors of n are the simplest way "CM physics" enters

Can get ϵ, dielectric function of a 'chunk' of H atoms

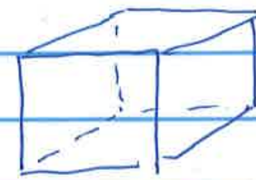
by doing the Phys 215 calculation of $\langle z \rangle$.

CM2

Such pictures are exceedingly crude since they treat solids as collections of independent atoms. Yet we know crystal structure, electrons break free to form a metal, etc.

Still n plays fundamental role.

Consider a metal as a box of e^- which are 'totally' free of their ions.



$L \times L \times L$ box
volume V

$$\psi(xyz) = \sqrt{\frac{2}{L}}\,\sin k_x x\,\sqrt{\frac{2}{L}}\,\sin k_y y\,\sqrt{\frac{2}{L}}\,\sin k_z z$$

$$k_x = \frac{n_x \pi}{L} \qquad k_y = \frac{n_y \pi}{L} \qquad k_z = \frac{n_z \pi}{L}$$

$$\psi(xyz) = \sqrt{\frac{8}{V}}\,\underbrace{\sin k_x x\,\sin k_y y\,\sin k_z z}$$

$$\underbrace{e^{i\vec{k}\cdot\vec{r}}} - \underbrace{e^{-i\vec{k}\cdot\vec{r}}}$$

usual plane wave states which have same energy

CM 3

Pauli principle : can only put $2e^-$ in any state (k_x, k_y, k_z)

Given N electrons must fill up states

$$N = \sum_{\substack{\text{occupied}\\ k}}^{k_{max}} 2 \rightarrow \frac{V}{(2\pi)^3}\int_0^{k_{max}} d^3k \; 2$$

$k_{max} \leftarrow k_{Fermi}$

$$\frac{N}{V} = \frac{2}{8\pi^3}\frac{4}{3}\pi k_F^3$$

$$k_F = (3\pi^2 n)^{1/3}$$

our friend n is relevant to metals too!

Apply $H^{(1)}$ is tricky if many e^- around because $H^{(1)}$ might try to push e^- into a state already occupied by another e^- "Pauli blocking"

Lindhardt dielectric function Apply $\vec{E}$ which has $\vec{k}$ and ω dependence.