

Green's function Review

A) Allow soln of PDE with "source term"

$$-\nabla^2 \phi(r) = \rho(r) / \epsilon_0$$

3D Poisson
Eqn

$G(\vec{r}-\vec{r}')$

$$\nabla^2 \left[-\frac{1}{4\pi |\vec{r}-\vec{r}'|} \right] = \delta(\vec{r}-\vec{r}')$$

$$\rightarrow \phi(r) = \frac{1}{4\pi\epsilon_0} \int d^3r' \frac{\rho(r')}{|\vec{r}-\vec{r}'|}$$

B) Allow soln of PDE w/o "source term" but with knowledge of initial conditions (bdy conditions)

i) 2D Laplace eqn $\nabla^2 \phi(x,y) = 0$

$$\phi(x,y) = \int_{-\infty}^{\infty} dx' \phi(x',0) \frac{y}{\pi(y^2 + (x-x')^2)}$$

$G(x-x', y)$

ii) 1D Wave eqn $0 < x < L$

$$y(x,t) = \int dx' G(x,x',t) y(x',t=0)$$

not $x-x'$ because boundaries

iii) Diffusion Eqn ...

iv) 3D Laplace Eqn (HW)



$\phi(x,y, z=0)$

GR2

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\uparrow$$

$$\vec{\nabla} \times \vec{A}$$

$$\vec{J} = \sigma \vec{E}$$

$$\uparrow$$

$$-\partial \vec{A} / \partial t$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu_0 \sigma (-\partial \vec{A} / \partial t)$$

$$-\nabla^2 \vec{A} + \nabla(\vec{\nabla} \cdot \vec{A}) = -\mu_0 \sigma \partial \vec{A} / \partial t$$

Starting
pt of
6.3

$$\leadsto \quad \partial \vec{A} / \partial t = \frac{1}{\mu_0 \sigma} \nabla^2 \vec{A} \quad \leftarrow \text{Diffusion Eqn}$$

Physics as \vec{A} changes in time

it produces an \vec{E} field

which produces a current \vec{J}

which produces a \vec{B} field $\leadsto \vec{A}$

Diffusive because $\vec{J} = \sigma \vec{E}$

represents flow with friction

(\vec{J} is uniform even though there is a constant push \vec{E} on charge carriers)