PHYSICS 200C, SPRING 2017
ELECTRICITY AND MAGNETISM

Assignment Four, Due Friday, May 19, 5:00 pm.

[1.] Consider two equal and opposite charges $\pm q$ moving in a circle in the xy plane or radius a at an angular velocity ω. As they move, the charges are always 180° apart on the circle. The motion is nonrelativistic, $v = \omega a << c$. Show that far away from the charges (distance to observer $R >> a$),

$$\vec{\mathbf{E}} = -\frac{qa\omega^2}{2\pi\epsilon_0 c^2 R}\,\hat{\mathbf{R}} \times \Big(\hat{\mathbf{R}} \times \big(\cos\omega t_r\,\hat{\mathbf{x}} + \sin\omega t_r\,\hat{\mathbf{y}}\big)\Big)$$

where $t_r = t - R/c$.

[2.] Find the total power radiated in problem [1].

[3.] Consider a dipole made up of a positive and a negative charge oscillating about the origin along the z-axis. That is, the position of the positive charge is $x_+ = y_+ = 0$ and $z_+ = (d/2)\cos\omega t$. the position of the negative charge is $x_- = y_- = 0$ and $z_- = -(d/2)\cos\omega t$. Calculate the electric field far away (distance to observer $R >> d$).

[4.] Which of the following expressions for a plane electromagnetic wave in vacuum are correct? Explain.

(a) $\vec{\mathbf{E}} = E_0 \sin(kz - \omega t)\,\hat{\mathbf{x}}$ $\qquad$ $\vec{\mathbf{B}} = E_0/c\,\cos(kz - \omega t)\,\hat{\mathbf{y}}$

(b) $\vec{\mathbf{E}} = E_0 \sin(kz - \omega t)\,\hat{\mathbf{y}}$ $\qquad$ $\vec{\mathbf{B}} = -E_0/c\,\sin(kz + \omega t)\,\hat{\mathbf{x}}$

(c) $\vec{\mathbf{E}} = E_0 \sin^2(kz - \omega t)\,\hat{\mathbf{x}}$ $\qquad$ $\vec{\mathbf{B}} = -E_0/c\,\sin^2(kz - \omega t)\,\hat{\mathbf{y}}$

(d) $\vec{\mathbf{E}} = E_0 \cos(ky - 2\omega t)\,\hat{\mathbf{x}}$ $\qquad$ $\vec{\mathbf{B}} = -E_0/c\,\cos(ky - 2\omega t)\,\hat{\mathbf{z}}$

(e) $\vec{\mathbf{E}} = E_0 \sin(ky + \omega t)\,(\hat{\mathbf{y}} + \hat{\mathbf{x}})$ $\qquad$ $\vec{\mathbf{B}} = \sqrt{2}E_0/c\,\sin(ky + \omega t)\,\hat{\mathbf{z}}$

[5.] The electric field of a plane wave is given by

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = E_0\,\Big(\cos(kx - \omega t)\,\hat{\mathbf{z}} + \sin(kx - \omega t)\,\hat{\mathbf{y}}\Big)$$

(a) Give the accompanying $\vec{\mathbf{B}}$ field.
(b) Describe the polarization of the wave. Accompany your answer with a usefully labelled diagram.
(c) Evaluate the time-averaged energy and momentum densities.

[6.] In the Gaussian laser beam discussed in class, find the equation of a ray that lies in the xz plane and passes through the waist as a distance x_0 from the axis of the beam.